

Nearing extremal intersecting giants and new decoupled sectors in $\mathcal{N} = 4$ SYM

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ABSTRACT: We study near-horizon limits of near-extremal charged black hole solutions to five-dimensional $U(1)^3$ gauged supergravity carrying two charges, extending the recent work of Balasubramanian et al. [1]. We show that there are two near-horizon decoupling limits for the near-extremal black holes, one corresponding to the near-BPS case and the other for the far from BPS case. Both of these limits are only defined on the $10d$ IIB uplift of the $5d$ black holes, resulting in a decoupled geometry with a six-dimensional part (conformal to) a rotating $BTZ \times S^3$. We study various aspects of these decoupling limits both from the gravity side and the dual field theory side. For the latter we argue that there should be two different, but equivalent, dual gauge theory descriptions, one in terms of the $2d$ CFT's dual to the rotating BTZ and the other as certain large R -charge sectors of $d = 4$, $\mathcal{N} = 4$ $U(N)$ SYM theory. We discuss new BMN-type sectors of the $\mathcal{N} = 4$ SYM in the $N \rightarrow \infty$ limit in which the engineering dimensions scale as $N^{3/2}$ (for the near-BPS case) and as N^2 (for the far from BPS case).

KEYWORDS: Black Holes in String Theory, AdS-CFT Correspondence.

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1. Introduction and summary

According to AdS/CFT conjecture [2, 3] any state/physical process in the asymptotically $AdS_5 \times S^5$ geometry can be described by a (perturbative) deformation of $\mathcal{N} = 4$, $d = 4$ supersymmetric Yang-Mills (SYM) theory. A class of deformations of $AdS_5 \times S^5$ are solutions to $\mathcal{N} = 2$, $d = 5$ $U(1)^3$ gauged supergravity (the “gauged STU model”), for a review e.g. see [4, 5]). Among these solutions there are geometries carrying charges under some (or all) of the three $U(1)$ ’s. These are generically $5d$ black hole type solutions. It is possible to uplift these solutions to $10d$ and obtain the corresponding type IIB solutions which are constant dilaton solutions only involving metric and the (self-dual) five-form field of IIB theory. These solutions which have been extensively studied from the gravity viewpoint (e.g. see [5] and references therein) can be 1/2, 1/4, 1/8 BPS respectively preserving 16, 8, 4 supercharges. The $10d$ BPS solutions have been called superstars [6].

In the $10d$ picture the 1/2 BPS solutions correspond to smeared (delocalized) spherical D3-branes [6], the giant gravitons [7]. These are branes wrapping a three sphere inside the S^5 part of the background $AdS_5 \times S^5$ geometry while moving on a geodesic along an $S^1 \in S^5$ transverse to the worldvolume S^3 and smeared (delocalized) over the remaining direction. The 1/2 BPS solutions are specified by a single parameter, the value of the charge. In a similar manner the two-charge 1/4 BPS and three-charge 1/8 BPS solutions can be understood as geometries corresponding to intersecting giant gravitons. The non-supersymmetric cases then correspond to turning on specific open string excitations on the supersymmetric (intersecting) giant gravitons.

Besides the (excited intersecting) spherical brane picture the $5d$ charged black hole type solutions should also have a description in the $\mathcal{N} = 4$ SYM on $R \times S^3$. The 1/2 BPS case is described by chiral primary operators in the subdeterminant basis [9]. In a similar fashion less BPS solutions correspond to operators involving two or three complex scalars in the $\mathcal{N} = 4$ vector multiplet [10]. The non-supersymmetric configurations when the solution is near-BPS (i.e. when $\frac{\Delta-J}{J} \ll 1$, where Δ is the scaling dimension and J is the R -charge of the corresponding operators) then correspond to insertion of “impurities” in the subdeterminant operators [11–13].

In this paper we intend to extend and elaborate further on the discussions of [1] and focus on the two-charge solutions. Noting that for these solutions we have a simple interpretation in terms of intersecting giants we pose the following question: Is there a limit in which the (low energy effective) gauge theory residing on the intersecting spherical brane system decouples from the bulk? In this paper we argue, by gathering supportive evidence from various sides, that the answer to this question is positive. As the first and very suggestive piece of evidence we show that there exist *two* such near-horizon, near-extremal limits, one corresponding to the *near-BPS* case and the other to the *far from BPS* case. In both cases there is an $X_{M,J} \times S^3$ geometry where $X_{M,J}$ is a global AdS_3 or an AdS_3 with conical singularity or a (rotating) BTZ black hole.

We use the existence of these decoupled geometries and the appearance of AdS_3 factors to argue that string theory on both of these decoupled backgrounds should have a description in terms of a $2d$ CFT, which is living on the intersection of two sets of spherical

D3-branes, intersecting on an S^1 (cross time). Recalling that the geometry we start with is an asymptotically $AdS_5 \times S^5$ space-time we expect to also have a description in terms of $\mathcal{N} = 4$ SYM on $R \times S^3$. Explicitly, there should be a sector (sectors) of $\mathcal{N} = 4$ SYM which is effectively described by a $2d$ gauge theory. We identify both the $2d$ gauge theory and the corresponding sector in $\mathcal{N} = 4$ SYM for the near-BPS decoupling limit. For the non-BPS decoupling limit we identify the corresponding sector in $\mathcal{N} = 4$ SYM and discuss properties of our conjectured $2d$ dual CFT.

This paper is organized as follows. In section 2, after reviewing the $5d$ SUGRA charged black hole solutions, we identify two extremal cases, the BPS solution and the non-BPS black hole solution which has a null singularity. We then focus on the two-charge case, turn on a “small” non-extremality parameter and take the “near-horizon” limit about these two extremal solutions to obtain two decoupled geometries containing $AdS_3 \times S^3$ factors. In the BPS case our solution is either supersymmetric or is a deformation of a supersymmetric background and the deformation parameter can be continuously tuned to zero. This case was discussed to some extent in [1] but for completeness we have included it. In the non-BPS case, the solution obtained after taking the limit is far from being BPS.

In section 3, we compute the (Bekenstein-Hawking) entropy of the corresponding $5d$ black hole and compare it against the same entropy for the $3d$ black hole and find an exact matching for both the BPS and the non-BPS cases. We take this matching as an evidence for the fact that in both cases we have a decoupled theory. This is of particular significance especially for the non-BPS case. In this section we make both of the near-horizon, near-extremal limits of previous section more precise by imposing the conditions under which one can trust the classical gravity description of the geometry obtained after the limit.

In section 4, we discuss a novel *consistent* reduction of $10d$ IIB SUGRA to a six-dimensional (super)gravity theory which besides the metric, involves a two-form and a scalar field with a non-trivial potential. Moreover, we also examine the Sen’s entropy function method [14] for computing the black hole entropy in $10d$, $5d$ as well as $6d$ and $3d$ viewpoints. We show that the $10d$ giant gravitons appear as strings, source of the two-form field, in this reduced $6d$ theory.

In section 5, we show that one can turn on the third charge in a perturbative manner (keeping the third charge much smaller than the other two). In this way, repeating the near-horizon, near-extremal limits on these three-charge geometries we obtain a rotating BTZ black hole. We again have two options, taking the near-horizon limit on the near-BPS solution or on the non-BPS, solution. We study the associated Bekenstein-Hawking entropy of these solutions from $5d$ and $3d$ viewpoints and show that, similarly to the static case, we obtain exactly the same result for the entropies.

In section 6, we elaborate on the $2d$ and $4d$ dual gauge theory descriptions of the decoupled near-horizon geometries for both of the near-BPS and far from BPS cases. The $2d$ dual gauge theory for the near-BPS case is closely related to standard the D1-D5 systems upon two T-dualities, as in this case the radius of the spherical giant three-branes are scaled to infinity and hence we are essentially dealing with two stacks of intersecting D3-branes with worldvolume $R \times S^1 \times T^2$ [1]. In the $4d$ language taking the near-horizon near-BPS limit corresponds to $N \rightarrow \infty$, $g_{\text{YM}}^2 = \text{fixed}$ limit and working with the sector of operators

carrying two R -charges, with both of the R -charges and the scaling dimension Δ of order $N^{3/2}$, while $\Delta - \sum_i J_i \sim N$. This is a generalization of the BMN limit [15] to the two-charge case. The far from BPS case, however corresponds to a different sector of the $\mathcal{N} = 4$ SYM; to the sector which is far from being BPS and in which the scaling dimension and the R -charges are of order N^2 while taking $N \rightarrow \infty$ and a certain combination of Δ and J^2 scales as N . For the near-extremal decoupled geometry, we argue that there should be a $2d$ dual CFT description and identify the central charge and discuss some other properties of this conjectured $2d$ CFT.

In the last section we give a summary of our results, outlook and discuss interesting open questions. In two appendices we have gathered some useful computations and conventions. In appendix A, we show the computations proving the consistency of the reduction of the $10d$ IIB theory to the $6d$ theory discussed in section 4. In appendix B, we give a concise review and fix conventions we use for the rotating BTZ and conical AdS_3 spaces.

2. Decoupling limits of near-extremal $5d$ black holes

In this section after reviewing the charged black hole solutions to five-dimensional $U(1)^3$ gauged supergravity, and their uplift to $10d$ IIB theory, we present two different near-horizon decoupling limits over the near-extremal black holes carrying two charges, one for the near-BPS solution and the other for far from BPS configuration.

2.1 Charged black hole solutions in $5d$

The black hole solutions that we consider in this paper were first obtained in the five-dimensional context in [16, 17]. They are static charged solutions to $\mathcal{N} = 2$ $U(1)^3$ gauged supergravity in five dimensions and hence are black hole solutions in the AdS_5 background. These solutions can be uplifted to ten dimensions as black hole (black-brane) deformations to $AdS_5 \times S^5$ [4] (see [5] for a review). We will first review the ten-dimensional black-brane solution.¹ The metric takes the form,

$$ds_{10}^2 = \sqrt{\Delta} ds_5^2 + \frac{1}{\sqrt{\Delta}} d\Sigma_5^2 \tag{2.1}$$

where

$$ds_5^2 = -\frac{f}{H_1 H_2 H_3} dt^2 + \frac{dr^2}{f} + r^2 d\Omega_3^2 \tag{2.2a}$$

$$d\Sigma_5^2 = \sum_{i=1}^3 L^2 H_i \left(d\mu_i^2 + \mu_i^2 [d\phi_i + a_i dt]^2 \right). \tag{2.2b}$$

¹We will follow the equations of [18], which corrects a typo in [4].

$(H_1 H_2 H_3)^{1/3} ds_5^2$ is the line element for the corresponding charged $5d$ black hole and $d\Sigma_5^2$ is the metric for a deformed S^5 . In the above $d\Omega_3^2$ is the round-metric on the unit S^3 and

$$H_i = 1 + \frac{q_i}{r^2}, \quad a_i = \frac{\tilde{q}_i}{q_i} \frac{1}{L} \left(\frac{1}{H_i} - 1 \right), \quad (2.3a)$$

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L^2} H_1 H_2 H_3, \quad \Delta = H_1 H_2 H_3 \left[\frac{\mu_1^2}{H_1} + \frac{\mu_2^2}{H_2} + \frac{\mu_3^2}{H_3} \right], \quad (2.3b)$$

$$\mu_1 = \cos \theta_1, \quad \mu_2 = \sin \theta_1 \cos \theta_2, \quad \mu_3 = \sin \theta_1 \sin \theta_2. \quad (2.3c)$$

As can be readily seen the ten-dimensional solutions asymptote (i.e. as $r \rightarrow \infty$) to $AdS_5 \times S^5$ where the radii of both of the AdS_5 and the S^5 are L . The S^5 is parameterized with the angles $\theta_1, \theta_2, \phi_1, \phi_2, \phi_3$. In terms of the $5d$ $U(1)^3$ gauged SUGRA the three gauge fields are given by the a_i (2.3a) [4].

The above metric represents a solution to $10d$ type IIB SUGRA with constant dilaton and with the following RR four-form gauge field

$$B_4 = -\frac{r^4}{L} \Delta dt \wedge d^3\Omega - L \sum_{i=1}^3 \tilde{q}_i \mu_i^2 \left(L d\phi_i - \frac{q_i}{\tilde{q}_i} dt \right) \wedge d^3\Omega, \quad (2.4)$$

where $d^3\Omega$ is the volume form on the unit three-sphere. The physical five-form field strength is obtained as

$$\mathcal{F}_5 = F_5 + *F_5, \quad F_5 = dB_4.$$

As $5d$ black holes the above solutions are identified with the physical ADM mass M and charges \tilde{q}_i which in terms of parameters of the solution μ and q_i are given by [19]^{2,3}

$$\tilde{q}_i = \sqrt{q_i(\mu + q_i)} \quad (2.5a)$$

$$M = \frac{\pi}{4G_N^{(5)}} \left(\frac{3}{2}\mu + q_1 + q_2 + q_3 + \frac{3L^2}{8} \right), \quad (2.5b)$$

where $G_N^{(5)}$ is the five-dimensional Newton constant and is related to the ten-dimensional one as

$$G_N^{(5)} = G_N^{(10)} \frac{1}{\pi^3 L^5}. \quad (2.6)$$

The last term in the ADM mass expression (2.5b) is the Casimir energy coming due to the fact that the global AdS_5 background has a compact $R \times S^3$ boundary. μ is a parameter which measures deviation from being BPS. For $\mu = 0$ case, $\tilde{q}_i = q_i$ and hence ADM mass up to the Casimir energy and factor of $\pi/4G_N^{(5)}$ is equal to the sum of the physical charges and therefore the solution is BPS. The BPS configuration with n number of non-vanishing q_i 's ($n = 1, 2, 3$) generically preserves $1/2^n$ of the 32 supercharges of the $AdS_5 \times S^5$ background, except for the three-charge case with $q_1 = q_2 = q_3$ which is $1/4$ BPS and corresponds to

²We would like to thank Alex Buchel, Mirjam Cvetič and Wafic Sabra for useful correspondence on the notion of (ADM) mass in the AdS backgrounds for gauged STU supergravity models.

³See also [20] for a general discussion on the relation between the ADM mass and charge in the holographic setting.

a 5d AdS-Reissner-Nordstrom black hole [21]. All the supersymmetric BPS solutions have naked singularity.

Black holes with regular horizons can only occur when $\mu \neq 0$ and hence are all non-supersymmetric. For the $\mu \neq 0$ cases depending on the number of non-zero charges, which can be one, two or three, we have different singularity and horizon structures [1, 6, 17].

As ten-dimensional IIB solutions, these black holes correspond to (smeared or delocalized) stack of intersecting spherical three-brane giant gravitons wrapping different $S^3 \in S^5$. The angular momentum that each stack of giants carries is [6]

$$J_i = \frac{\pi L}{4G_N^{(5)}} \tilde{q}_i . \tag{2.7}$$

The number of branes in each stack is then given by [6]

$$N_i = \frac{2J_i}{N} = \frac{\pi^4}{2N} \cdot \frac{L^8}{G_N^{(10)}} \cdot \frac{\tilde{q}_i}{L^2}, \tag{2.8}$$

which could be understood noting that each giant, being a D3-brane and obeying the DBI action, is carrying one unit of the RR charge in units of three-brane tension $T_3 = 1/(8\pi^3 l_s^4 g_s)$.

Here we give a short review of cases with different number of charges.

- *One-charge black hole.* At $\mu = 0$ we have a null nakedly singular solution which preserves 16 supercharges. As soon as we turn on μ the solution develops a horizon with a space-like singularity sitting behind the horizon.

The ten-dimensional IIB uplift of these solutions contain non-trivial five-form flux and correspond to various giant graviton configurations [6]. The one charge case with $\mu = 0$ corresponds to 1/2 BPS three sphere giant configuration wrapping an S^3 inside the S^5 while moving with the angular momentum $J \propto q$. This gravity configuration, however, describes a giant smeared over (delocalized in) two directions inside S^5 transverse to the worldvolume of the brane. Turning on μ then corresponds to adding open string excitations to the giant graviton while keeping the spherical shape of the giant.

- *Two-charge black hole.* For $0 \leq \mu < \mu_c$ we have a time-like but naked singularity where $\mu_c = q_2 q_3 / L^2$. At $\mu = \mu_c$ we have an *extremal, but non-BPS* black hole solution with a zero size horizon area (horizon is at $r = 0$) and $r = 0$ in this case is a null naked singularity. As we increase μ from μ_c the solution develops a finite size horizon and the space-like singularity hides behind the horizon.

As ten-dimensional solutions, the two-charge case at $\mu = 0$ corresponds to two sets of delocalized giant gravitons wrapping two S^3 's inside S^5 while rotating on two different S^1 directions. The worldvolume of the giants overlap on a circle. If one of the charges is much smaller than the other one a better (perturbative) description of the system is in terms of a rotating single giant where as a result of the rotation the giant is deformed from the spherical shape. As in the single charge case, turning on

μ , especially when μ is small enough, corresponds to adding open string excitations while keeping the $U(1)$ symmetry of the giants intersection.

For the extremal case at $\mu = \mu_c$ the brane picture is more involved. In this case we are dealing with intersecting giants which are generically far from being BPS and effectively we are dealing with a stack of giants with worldvolume $R \times S^1 \times \Sigma_2$, where Σ_2 is a compact $2d$ surface inside the S^5 . Out of extremality, measured by $\mu - \mu_c$, then corresponds to excitations/fluctuations above this stack of giants. In the rest of this paper we will study dynamics of a class of these excitations.

- *Three-charge black hole.* For $0 \leq \mu < \mu_c$ we have a time-like naked singularity, the singularity is, however, behind $r = 0$ (one can extend the geometry past $r = 0$). At some critical μ , $\mu = \mu_c$, we have an extremal solution with a finite size horizon (function f has double zeros at some $r_h \neq 0$) [17]. For $\mu > \mu_c$ the geometry has two inner and outer horizons.

From the ten-dimensional viewpoint the three-charge case corresponds to a set of three smeared giant gravitons intersecting only on the time direction and the giants in each set moving on either of the three S^1 directions in the S^5 , which in (2.2b) are parameterized by ϕ_i . Again if one of the charges is much smaller than the other two a better description of the system is in terms of two giants intersecting on an S^1 , but the third charge appears as a rotation on the S^1 . We will return to this latter case in more detail in section 5.

2.2 The near-horizon limit of two-charge solutions

As discussed and reviewed in the previous subsection for the two and three-charge cases we have extremal black holes. These extremal black holes can be BPS or non-BPS. One may then expect that for both of these cases there should exist a “near-horizon” limit in which the theory on the corresponding intersecting giants decouple from the bulk. To study this we need to first analyze the near-horizon geometry for such extremal (or near-extremal) solutions. Although in this paper we mainly focus on the two-charge case, we discuss the three-charge case, when the third charge is much smaller than the other two, in section 5. We analyze and discuss both of the two-charge near-BPS and far from BPS cases in parallel. The near-BPS case has also been analyzed in [1].

To start the analysis let us choose the two non-vanishing charges to be q_2 and q_3 . In this case the function f in the metric takes the form

$$f = \frac{r^2}{L^2} + f_0 - \frac{\mu - \mu_c}{r^2}, \tag{2.9}$$

where

$$f_0 = 1 + \frac{q_2 + q_3}{L^2}, \tag{2.10}$$

and

$$\mu_c = \frac{q_2 q_3}{L^2}. \tag{2.11}$$

We use the 5d metric to locate the horizon, which occurs where g^{rr} vanishes, or at the roots of $r^{4/3}f$. From (2.9) it is evident that for $\mu = \mu_c$ we have a double zero at $r = 0$ (for $\mu < \mu_c$ f is positive definite and for $\mu > \mu_c$ f has a single positive root). Therefore, at $\mu = \mu_c$ we are dealing with an extremal black hole (or from 10d viewpoint, black-brane) solution in which both horizon and singularity are at $r = 0$.

The radius of the S^3 in the five-dimensional metric is proportional to $(H_1 H_2 H_3)^{1/3} r^2$. Only for the three-charge case, in the near-horizon limit $r \rightarrow r_h \neq 0$, we get a constant term [22]. For the two-charge case, the near-horizon limit $r \rightarrow 0$ gives $(q_2 q_3)^{1/3} r^{2/3}$ which is clearly not a product geometry. As we will show, however, the factorization happens if we take the limit over the 10d solution and this is what we do here.

2.2.1 Near-horizon limit, the near-BPS case

As argued the BPS case happens when $\mu = 0$. In the near-horizon limit we consider in this subsection, together with $r \rightarrow 0$ we also consider μ to be very small, explicitly we consider either of the following limits [1]

- $\mu_1 \sim 1$ case

$$\begin{aligned} r &= \epsilon \tilde{\rho}, & \mu_i &= \epsilon^{1/2} x_i, \\ \mu - \mu_c &= \epsilon^2 M, & q_i &= \epsilon \hat{q}_i, & i &= 2, 3, \end{aligned} \tag{2.12}$$

while keeping $\tilde{\rho}$, \hat{q}_i , M , x_i , ϕ_i , L fixed. Note also that, as $\mu_1^2 = 1 - \mu_2^2 - \mu_3^2$, in this limit $\mu_1 = 1 + \mathcal{O}(\epsilon^2)$. This limit corresponds to $\theta_1 \sim \epsilon^{1/2}$, $\theta_2 = \text{fixed}$ cf. (2.3c).

- $\mu_1 \sim \mu_1^0 \neq 1$ case

$$\begin{aligned} r &= \epsilon \tilde{\rho}, & \theta_i &= \theta_i^0 - \epsilon^{1/2} \hat{\theta}_i, & 0 &\leq \theta_i^0 \leq \pi/2, \\ \mu - \mu_c &= \epsilon^2 M, & q_i &= \epsilon \hat{q}_i, & \psi_i &= \frac{1}{\epsilon^{1/2}} \left(\phi_i - \frac{t}{L} \right), & i &= 2, 3, \end{aligned} \tag{2.13}$$

while keeping $\tilde{\rho}$, \hat{q}_i , M , θ_i^0 , x_i , L fixed.

As we can see in both of these cases

$$\gamma^2 \equiv \frac{\mu - \mu_c}{\mu_c} \tag{2.14}$$

is kept fixed, $\mu \sim \epsilon^2$ and hence the physical charges $\tilde{q}_i = q_i \sim \epsilon$.

Taking the limit we arrive at the $AdS_3 \times S^3 \times T^4$ geometry

$$ds^2 = \epsilon \left[R_S^2 (ds_{\text{AdS}}^2 + d\Omega_3^2) + \frac{L^2}{R_S^2} ds_{\mathcal{C}_4}^2 \right] \tag{2.15}$$

where

$$ds_{\text{AdS}}^2 = -(\rho^2 - \gamma^2) d\tau^2 + \frac{d\rho^2}{\rho^2 - \gamma^2} + \rho^2 d\phi_1^2 \tag{2.16}$$

with

$$\rho = \frac{L}{(\hat{q}_2 \hat{q}_3)^{1/2}} \frac{r}{\epsilon}, \quad \tau = \frac{1}{L} t.$$

The S^3 radius R_S^2 and the four-dimensional metric $ds_{\mathcal{C}_4}^2$ have different forms for the two cases:

- $\mu_1 \sim 1$ case

$$R_S^2 = \sqrt{\hat{q}_2 \hat{q}_3}, \quad ds_{\mathcal{C}_4}^2 = \sum_{i=2,3} \hat{q}_i (dx_i^2 + x_i^2 d\psi_i^2) \quad (2.17)$$

where $\psi_i = \phi_i - \frac{t}{L}$.

- $\mu_1 \sim \mu_1^0 \neq 1$ case

$$R_S^2 = \sqrt{\hat{q}_2 \hat{q}_3 \mu_1^0}, \quad ds_{\mathcal{C}_4}^2 = \sum_{i=2,3} \hat{q}_i (dx_i^2 + (\mu_i^0)^2 d\psi_i^2) \quad (2.18)$$

where $\mu_2^0 = \sin \theta_1^0 \cos \theta_2^0$, $\mu_3^0 = \sin \theta_1^0 \sin \theta_2^0$, $dx_2 = \cos \theta_1^0 \cos \theta_2^0 d\hat{\theta}_1$ and $dx_3 = \cos \theta_1^0 \sin \theta_2^0 d\hat{\theta}_1 + \cos \theta_2^0 \sin \theta_1^0 d\hat{\theta}_2$.

In either case the \mathcal{C}_4 part of the geometry after appropriate periodic identifications is describing a T^4 and hence the solutions are $AdS_3 \times S^3 \times T^4$. For $\gamma^2 = -1$ we have a global AdS_3 space, for $-1 < \gamma^2 < 0$ it is a conical space, for $\gamma^2 = 0$ we have a massless BTZ and for $\gamma^2 > 0$ we are dealing with a static BTZ black hole of mass γ^2 . (For more detailed discussion see appendix B.) These geometries are, upon two T-dualities, related to standard the D1-D5 system and the corresponding arguments are applicable to this case [1, 23]. A detailed discussion on the $AdS_3 \times S^3$ geometries and the spectrum of supergravity/string theory in $AdS_3 \times S^3$ compactification may be found in [24] and references therein. We will give a brief review in section 6.2.1.

2.2.2 Near-horizon limit, the far from BPS case

We take the following near-horizon decoupling limit over the *far from BPS* solution, while keeping μ_c fixed, i.e.

$$\begin{aligned} r &= \epsilon \tilde{\rho}, & t &= \frac{\tilde{\tau}}{\epsilon}, & \mu - \mu_c &= \epsilon^2 M \\ \phi_1 &= \frac{\varphi}{\epsilon}, & \phi_i &= \psi_i + \frac{\tilde{q}_i}{q_i L} \frac{\tilde{\tau}}{\epsilon}, & i &= 2, 3 \end{aligned} \quad (2.19)$$

where $\tilde{\rho}$, $\tilde{\tau}$, φ , ψ_i , M , L are kept fixed while taking $\epsilon \rightarrow 0$. Taking this limit we also keep q_i/L^2 and hence f_0 , μ_c/L^2 fixed.

In the above near-horizon near-extremal limit, the leading contribution from functions f , Δ , H_i appearing in (2.1) become

$$f = f_0 - \frac{M}{\tilde{\rho}^2}, \quad \Delta = \mu_1^2 \frac{q_2 q_3}{\tilde{\rho}^4} \cdot \frac{1}{\epsilon^4}, \quad H_i = \frac{q_i}{\tilde{\rho}^2} \cdot \frac{1}{\epsilon^2}. \quad (2.20)$$

The ten-dimensional metric (2.1) in the limit (2.19), after some redefinition of coordinates takes the form

$$ds_{10}^2 = \mu_1 (R_{AdS_3}^2 ds_3^2 + R_S^2 d\Omega_3^2) + \frac{1}{\mu_1} ds_{\mathcal{M}_4}^2 \quad (2.21)$$

where

$$ds_3^2 = -(\rho^2 - \rho_0^2) d\tau^2 + \frac{d\rho^2}{\rho^2 - \rho_0^2} + \rho^2 d\varphi^2, \quad (2.22)$$

$d\Omega_3^2$ is the metric for a round three sphere of unit radius and

$$ds_{\mathcal{M}_4}^2 = \frac{L^2}{R_S^2} [q_2 (d\mu_2^2 + \mu_2^2 d\psi_2^2) + q_3 (d\mu_3^2 + \mu_3^2 d\psi_3^2)]. \quad (2.23)$$

In the above

$$R_S^2 \equiv \sqrt{q_2 q_3} = \sqrt{L^2 \mu_c}, \quad R_{\text{AdS}_3}^2 = \frac{R_S^2}{f_0}, \quad (2.24a)$$

$$\rho_0^2 = \frac{M}{\mu_c}, \quad (2.24b)$$

and the new coordinates ρ and τ in terms of the original coordinates t, r are defined as⁴

$$\tau = \epsilon \frac{R_S}{R_{\text{AdS}_3}} \frac{t}{L}, \quad \rho = \frac{L}{R_S R_{\text{AdS}_3}} \frac{r}{\epsilon}. \quad (2.25)$$

Note that $\mu_1^2 = 1 - \mu_2^2 - \mu_3^2$ and therefore μ_1 is not a constant (in contrast to the near-BPS case). As we see after the decoupling limit the metric has taken the form of a six-dimensional part which is conformal to $AdS_3 \times S^3$ and a four-dimensional part conformal to \mathcal{M}_4 which is a Kähler manifold.

For $\rho_0^2 \geq 0$ the metric (2.22) describes a stationary BTZ black hole in a *locally*⁵ AdS_3 background of radius R_{AdS_3} (2.24a) and of mass ρ_0^2 (2.24b). For $\rho_0^2 < 0$, however, we have an AdS_3 with conical singularity and the deficit angle $2\pi(1 - \delta)$ where

$$\delta = \frac{\mu - \mu_c}{\mu_c} = \epsilon^2 \rho_0^2. \quad (2.26)$$

It is notable that the angle in the BTZ which is parameterized by φ is coming from the part which was in the S^5 part of the original $AdS_5 \times S^5$ background, while the rest of the six-dimensional part of metric come from the original AdS_5 geometry; the \mathcal{M}_4 is coming from the S^5 piece. As mentioned the angle φ is ranging over $[0, 2\pi\epsilon]$, nonetheless the causal boundary of the near-horizon decoupled geometry is still $R \times S^1$. To see this we note that at large, but fixed ρ the AdS_3 part of the metric takes the form

$$ds_3^2 \sim R_{\text{AdS}_3}^2 \epsilon^2 \rho^2 (-dt^2 + d\phi_1^2), \quad (2.27)$$

where t is the (global) time direction in the original AdS_5 geometry, and therefore the causal boundary of this space is $R \times S^1$.

Metric (2.21) is a constant dilaton solution to IIB SUGRA with the four-form field

$$B_4 = -L^2 (\tilde{q}_2 \mu_2^2 d\psi_2 + \tilde{q}_3 \mu_3^2 d\psi_3) \wedge d^3\Omega_3, \quad (2.28)$$

where in the near-horizon, far from BPS limit (2.19)

$$\tilde{q}_2^2 = q_2^2 \left(1 + \frac{q_3}{L^2}\right), \quad \tilde{q}_3^2 = q_3^2 \left(1 + \frac{q_2}{L^2}\right). \quad (2.29)$$

⁴This scaling is a generic feature of near-horizon, near-extremal limits, e.g. see [25].

⁵Note that the angle φ is ranging over $[0, 2\pi\epsilon]$.

The above four-form can be obtained from taking the decoupling limit (2.19) over the four-form of the original solution (2.4).

It is interesting to note the similarities between this decoupled geometry and the one given in (2.15)–(2.18). The geometry (2.21) is the much expected “global decoupled solution” of [1]. In (2.15) the radii of the AdS_3 and the S^3 are equal while in (2.21) they are different. Moreover, the range of the φ coordinate in (2.21) is $[0, 2\pi\epsilon]$ while that of ϕ_1 in (2.15) is $[0, 2\pi]$. As a side comment we note that, similarly to the original two-charge extremal solution, the geometry obtained in the near-horizon far from BPS limit, even when $\mu = \mu_c$, is not preserving any supersymmetries of the 10d IIB theory.

3. Entropy of the two-charge solution

In this section, we first compute the entropy of the 5d black hole and take both the near-horizon limits on it. Moreover, we argue how ϵ should scale with N in both cases. We then compute the entropy of the 3d BTZ black hole that is part of the geometry of the near-extremal near-horizon limit, and show that it precisely agrees with the 5d entropy in the same limit. This provides the first piece of evidence for the fact that this limit is indeed a nice decoupling limit.

3.1 Black hole entropy, 5d viewpoint

To compute the Bekenstein-Hawking entropy of the two-charge 5d black hole we recall that its metric is given by

$$ds^2 = -(H_2 H_3)^{-\frac{2}{3}} f dt^2 + (H_2 H_3)^{\frac{1}{3}} (f^{-1} dr^2 + r^2 d^2\Omega_3). \quad (3.1)$$

Zero(s) of $r^{4/3} f$ determine the location of the horizon(s). The area of the horizon is then

$$A_h^{(5)} = 2\pi^2 r_h^3 (H_2 H_3)^{1/2} |_{r=r_h}. \quad (3.2)$$

The Bekenstein-Hawking entropy is

$$S_{\text{BH}} = \frac{A_h^{(5)}}{4G_N^{(5)}}.$$

Recalling that $G_N^{(5)} = \frac{G_N^{(10)}}{\pi^3 L^5}$, and that

$$G_N^{(10)} = 8\pi^6 g_s^2 l_s^8, \quad L^4 = 4\pi g_s N l_s^4, \quad (3.3)$$

we obtain

$$S_{\text{BH}} = \frac{1}{2\pi} N^2 \cdot \frac{A_h^{(5)}}{L^3}. \quad (3.4)$$

As we see, once the area is measured in AdS_5 units L , the entropy generically scales like N^2 . However, one should remember that the area of the horizon also scales with ϵ and in fact in two different ways for the two near-horizon limits. Therefore, we discuss the near-BPS and far from BPS cases separately. Before that, we should stress that the notion

of black hole entropy is only valid when horizon area is not Planckian and we are in the regime we can trust classical gravity description, explicitly that is when

$$S_{\text{BH}} \gg 1, \quad \frac{G_N^{(10)}}{l_s^8} \cdot S_{\text{BH}} \gg 1 \quad (\text{or } g_s^2 \cdot S_{\text{BH}} \gg 1). \quad (3.5)$$

Moreover, one should ensure that all the curvature invariants remain small (in Planck or string units).

3.1.1 5d black hole entropy of near-BPS case

In the near-BPS limit the horizon is located at

$$r_h^2 = \mu - \mu_c \quad (3.6)$$

and hence

$$S_{\text{BH}}^{\text{Near-BPS}} = \pi \gamma \frac{\hat{\mu}_c}{L^2} N^2 \epsilon^2, \quad (3.7)$$

where γ is defined in (2.14), and $\hat{\mu}_c = \mu_c/\epsilon^2$. In this case the curvature components scale as $1/\epsilon$ (in units of L^{-2}). Validity of classical gravity arguments then implies that one should scale N to infinity as well: $N \sim \epsilon^{-\alpha}$, $\alpha \geq 1$. This consideration is, however, not strong enough to fix α . Noting the form of metric, that it has a factor of ϵ in front and that one expects the string scale to be the shortest physical length leads to

$$\epsilon \sim l_s^2 \Rightarrow N \sim \epsilon^{-2}. \quad (3.8)$$

Once the above scaling of ϵ and N is considered, we see that the entropy (3.7) scales as $N \sim \epsilon^{-2}$ to infinity.

As was argued in [1], only a certain class of massless open string modes on the intersecting giants survive the scaling (3.8). We will discuss in section 6 that these modes constitute the degrees of freedom of certain $2d$ CFT's.

In sum, our complete near-horizon, near-BPS limit is defined as an $\alpha' = l_s^2 \sim \epsilon \rightarrow 0$ limit together with (2.12) (or (2.13)), while keeping $L^4 \sim N l_s^4$ fixed.

3.1.2 5d black hole entropy of the far from BPS case

In the far from BPS limit of (2.19) to order ϵ , we have

$$r_h^2 = \frac{\mu - \mu_c}{f_0} + \mathcal{O}(\epsilon^4). \quad (3.9)$$

and hence

$$S_{\text{BH}}^{\text{Far from BPS}} = \pi \frac{\mu_c}{L^2} \cdot \frac{\rho_0}{\sqrt{f_0}} N^2 \epsilon. \quad (3.10)$$

To ensure (3.5) and also demanding the curvature components to remain small (in $10d$ string or Planck units) one should also send $N \rightarrow \infty$ while keeping ρ_0 and μ_c/L^2 finite. This is done if we scale $N \sim \epsilon^{-\beta}$, $\beta \geq \frac{1}{2}$. In our case, as we will discuss in section 4, $\beta = 1$ is giving the appropriate choice,

$$N \sim \epsilon^{-1} \rightarrow \infty. \quad (3.11)$$

In sum, in our limit we keep $L, g_s, q_i/L^2$ and ρ_0 finite while taking $l_s^4 \sim N^{-1} \sim \epsilon \rightarrow 0$. Similarly to the near-BPS case of section 3.1.1, in this case $S_{\text{BH}} \sim N \rightarrow \infty$.

3.2 The 3d BTZ black hole entropy, the far from BPS case

To work out the Bekenstein-Hawking entropy of the BTZ black hole obtained after taking the limit we should have the relevant three dimensional Newton constant. To this end, we show that there is a consistent reduction of the 10d IIB theory over \mathcal{M}_4 to a six-dimensional (super)gravity theory. Computations showing the consistency of the reduction are given in the appendix A. The Newton constant of this six-dimensional theory is (A.5)

$$G_N^{(6)} = \frac{G^{(10)}}{\frac{\pi^2}{2}L^4} = \frac{\pi^2 L^4}{N^2}, \tag{3.12}$$

and its action is given in (A.18).

As will be shown in the next section, the geometry we obtain after taking the limit is BTZ $\times S^3$ solution to this 6d theory. One can hence make a further reduction of this 6d theory on the S^3 to obtain a 3d gravity theory. Similarly to the standard case which e.g. was discussed in [23], this 3d (gauged super-)gravity has $SL(2, R)^2$ gauge symmetry (for the pure gravity) and a gauge group which has $U(1)_L \times U(1)_R$ as its sub-group. Noting that in our case the radius of S^3 is R_S , the corresponding 3d Newton constant is

$$G_N^{(3)} = \frac{G_N^{(6)}}{2\pi^2 R_S^3} = \frac{G_N^{(10)}}{\pi^4 L^4 R_S^3} = \left(2N^2 \cdot \frac{R_S^3}{L^4}\right)^{-1}. \tag{3.13}$$

The Bekenstein-Hawking entropy of the corresponding BTZ is then given by

$$S_{\text{BH}}^{(3)} = \frac{A^{(3)}}{4G_N^{(3)}}$$

where $A^{(3)}$ is the area of horizon for the BTZ black hole. For the far from BPS case that is

$$A^{(3)} = 2\pi\epsilon R_{\text{AdS}_3} \rho_0. \tag{3.14}$$

In computing the area of the horizon of the BTZ black hole (3.14) one should recall that φ which parameterizes the horizon circle is ranging from 0 to $2\pi\epsilon$ (2.19). Therefore,

$$S_{\text{BH}}^{(3)} = \pi \frac{R_{\text{AdS}} R_S^3}{L^4} \rho_0 N^2 \epsilon, \tag{3.15}$$

which is exactly the same as (3.10) once we recall that $R_{\text{AdS}} = R_S/\sqrt{f_0}$ and that $\mu_c = R_S^4/L^2$.

The exact matching of the entropies of the 5d black hole and that of the 3d BTZ is a strong sign of the fact that in the decoupling *far from BPS*, near-horizon limit we have taken we have not lost any degrees of freedom.⁶ This brings the hope that despite the lack of supersymmetry we may still look for a dual 1 + 1 dimensional gauge theory descriptions. We will return to this point in section 6.

⁶It is noteworthy that the horizon of the 5d black hole is $S^3 \in AdS_5$ while that of the 3d BTZ black hole is an $S^1 \in S^5$ in the original $AdS_5 \times S^5$ geometry and hence is not present in the 5d black hole picture.

4. The 6d analysis of the far from BPS solution

In previous sections we discussed two near-horizon decoupling limits of the two-charge 10d black-brane solutions. For the near-BPS case it is immediate to check that the $AdS_3 \times S^3 \times \mathcal{C}_4$ obtained after the limit is again a solution to IIB theory. This is not, however, obvious for the far from BPS case. In this section we discuss this issue through running (and in fact generalizing and extending) Sen's entropy function method [14] for the 6d BTZ $\times S^3$ geometry. In appendix A, we show that there is a consistent reduction of the 10d IIB theory to a 6d theory described in (A.18); hence showing that the BTZ $\times S^3$ is a solution to this 6d theory is enough to guarantee that the 10d near-horizon far from BPS geometry is a solution to IIB theory.

In addition, using the entropy function method we compute the entropy of the near-extremal BTZ $\times S^3$ solution as the near-horizon limit of a (extremal) black string solution of this 6d theory and show that this entropy is exactly equal to the entropy of the 5d black hole computed in section 3.1.2. This is very suggestive that one may use this 6d (black) string picture to identify the dynamical degrees of freedom of the dual 2d CFT description.

To run the entropy function machinery we start with our 6d (super)gravity action (A.18):

$$S = \frac{1}{16\pi G_N^{(6)}} \int dx^6 \sqrt{-g^{(6)}} \left[R^{(6)} - g^{\mu\nu} \frac{\nabla_\mu X \nabla_\nu X}{X^2} + \frac{4}{L^2} \left(X + \frac{1}{X} \right) - \frac{1}{3} X^2 F_{\mu\nu\rho} F^{\mu\nu\rho} \right]. \quad (4.1)$$

The next step is the near-horizon field configuration, which for extremal black holes (and black strings) is an $AdS_m \times S^n$ geometry. In our case, however, we have a BTZ $\times S^3$ solution. Recalling that the Riemann curvature for a (rotating) BTZ black hole (which is the most general 3d black hole geometry) is the same as the Riemann curvature for AdS_3 , we may use the standard steps of the usual entropy function method. We will comment more on this issue in the discussion section.

The ansatz for the field configuration is

$$ds^2 = v_1 \left(-(\rho^2 - \rho_0^2) d\tau^2 + \frac{d\rho^2}{\rho^2 - \rho_0^2} + \rho^2 d\varphi^2 \right) + v_2 d\Omega_3^2 \quad (4.2a)$$

$$F = e \text{vol}(AdS_3) + p \text{vol}(S^3) \quad (4.2b)$$

$$X = u = \text{const}, \quad (4.2c)$$

where $\text{vol}(AdS_3)$ and $\text{vol}(S^3)$ are respectively the volume forms of AdS_3 (or BTZ black hole) and the three sphere of unit radius. v_1 , v_2 , e and u are constants to be determined by the equations of motion in terms of the electric charge Q_e ,

$$Q_e \equiv \frac{1}{8\pi^2} \int_{S^3} \frac{\partial \left(\sqrt{-g^{(6)}} \mathcal{L} \right)}{\partial F_{\tau\rho\varphi}} d^3\Omega \quad (4.3)$$

where the above integration over an S^3 of unit radius, and the magnetic charge Q_m which is equal to p . The equations governing these parameters are obtained from variation of the

entropy function F which is defined as

$$F(v_i, e, u; Q_e, p = Q_m) = \frac{1}{16G_N^{(6)}} \int dx^H \sqrt{-g^{(6)}} \left(\frac{1}{2} F_{\tau\mu\nu} \frac{\partial \mathcal{L}}{\partial F_{\tau\mu\nu}} - \mathcal{L} \right), \quad (4.4)$$

the $\frac{1}{2}$ factor has appeared because we are dealing with a two-form field and $\{x^H\}$ is the four-dimensional horizon of the $6d$ (presumably) black string solution. For us and in metric (4.2a), this is $S^1 \times S^3$ where S^1 is a circle of radius ρ_0 parameterized by $\varphi \in [0, 2\pi\epsilon]$.⁷

Plugging the ansatz (4.2) into (4.4) we find

$$F(v_i, e, u; Q_e, Q_m) = \frac{\pi^3 \rho_0 \epsilon}{G_N^{(6)}} \left(e Q_e - \frac{1}{4} (v_1 v_2)^{\frac{3}{2}} \left[\frac{6}{v_2} - \frac{6}{v_1} + \frac{4}{L^2} \left(u + \frac{1}{u} \right) \right] + \frac{1}{2} u^2 \left[Q_m^2 \left(\frac{v_1}{v_2} \right)^{\frac{3}{2}} - e^2 \left(\frac{v_2}{v_1} \right)^{\frac{3}{2}} \right] \right). \quad (4.5)$$

Field equations which give values of v_i, u, e in terms of electric and magnetic charges Q_e and Q_m are

$$\frac{\partial F(v_i, e, u; Q_e, Q_m)}{\partial \Phi_I} = 0, \quad \Phi_I = \{v_i, e, u\} \quad (4.6)$$

After some simplifications the above equations take the form

$$\frac{1}{v_2} - \frac{1}{v_1} = -\frac{1}{L^2} \left(u + \frac{1}{u} \right), \quad (4.7a)$$

$$\frac{1}{v_2} + \frac{1}{v_1} = u^2 \left(\frac{e^2}{v_1^3} + \frac{Q_m^2}{v_2^3} \right), \quad (4.7b)$$

$$\frac{1}{L^2} \left(u - \frac{1}{u} \right) = -u^2 \left(\frac{e^2}{v_1^3} - \frac{Q_m^2}{v_2^3} \right), \quad (4.7c)$$

$$Q_e = \left(\frac{v_2}{v_1} \right)^{\frac{3}{2}} u^2 e. \quad (4.7d)$$

It is readily seen that

$$v_2 = \sqrt{q_2 q_3}, \quad v_1 = \frac{\sqrt{q_2 q_3}}{1 + \frac{q_2 + q_3}{L^2}}, \quad u = \sqrt{\frac{q_2}{q_3}}, \quad (4.8)$$

is a solution to (4.7) provided that

$$Q_e^2 \equiv q_2^2 \left(1 + \frac{q_3}{L^2} \right), \quad Q_m^2 \equiv q_3^2 \left(1 + \frac{q_2}{L^2} \right). \quad (4.9)$$

In order to match our notations and conventions with those of previous sections we choose:

$$Q_e \equiv \tilde{q}_2, \quad Q_m \equiv \tilde{q}_3. \quad (4.10)$$

⁷It should be noted that gravity equations of motion, and hence the entropy function method, are local differential equations and are hence blind to the range of coordinates e.g. the φ direction.

The above provides a crosscheck that the $BTZ \times S^3$ geometry obtained is indeed a solution to our $6d$ theory and hence the $10d$ IIB theory. Moreover, it makes a direct connection between the $6d$ charges Q_e, Q_m and the number of five-form fluxes (and hence number of giants) in the $10d$ setting.

As discussed in section 2, in the $10d$ setting in the two-charge case we are dealing with a system of intersecting giant three-sphere branes consisting of $N\tilde{q}_2/2L^2$ and $N\tilde{q}_3/2L^2$ giants which are intersecting over a circle (the φ direction in our $BTZ \times S^3$ solution). In the $6d$ setting, however, we are dealing with a system of strings along φ direction which are electrically and magnetically charged under the three-form F_3 . These (circular) strings are the giant three-branes wrapping two cycles of the four-dimensional reduction manifold \mathcal{M}_4 , one set of them are wrapping μ_2, ψ_2 directions and the other μ_3, ψ_3 directions. The tension of the $6d$ strings are then

$$T_s^{(6)} = T_3(\pi L^2) = \frac{N}{2\pi L^2}, \tag{4.11}$$

where we have used $T_3^{-1} = (2\pi)^3 l_s^4 g_s$, $L^4 = 4\pi l_s^4 g_s N$ and that the volume of the two cycles over which the three-branes are wrapping, are πL^2 (see also [26] for a similar discussion). It is also notable that

$$T_s^{(6)} = \frac{1}{2\sqrt{G_N^{(6)}}}. \tag{4.12}$$

In the near-horizon far from BPS limit as discussed in previous sections, we are taking $N \rightarrow \infty$ and this is done in a such a way that (*cf.* (3.11)) $T_s^{(6)} \sim \epsilon^{-1}$. In fact the choice for the scaling of N with respect to ϵ , (3.11), was made requiring, similarly to usual near-horizon decoupling limits e.g. see [2, 3], the $6d$ string length squared $1/2\pi T_s^{(6)}$ to scale as ϵ .⁸

Assuming that each string carries one unit of electric or magnetic charge,⁹ the number of electrically or magnetically charged strings are hence

$$N_e = 2\pi T_s^{(6)} Q_e = N \frac{\tilde{q}_2}{L^2}, \quad N_m = 2\pi T_s^{(6)} Q_m = N \frac{\tilde{q}_3}{L^2}, \tag{4.13}$$

⁸Although the arguments of this section are mainly made having the far from BPS case in mind, for the near-BPS case one may give a similar $6d$ description. The $AdS_3 \times S^3$ geometry obtained as near-BPS, near-horizon limit is a solution to a $6d$ SUGRA in which there is no potential term for the scalar X , e.g. see [24]. In this case the $AdS_3 \times S^3$ solution is the near-horizon limit of electrically and magnetically charged $6d$ strings. From the $10d$ IIB viewpoint, however, this corresponds to intersecting giants, which have two of their directions on the $4d$ compact part \mathcal{C}_4 [1]. From this one may read the effective $6d$ string tension. Recalling the form of metric after the decoupling limit (2.15) we have

$$T_s^{(6)}|_{Near\ BPS} \sim T_3 \epsilon L^2 \sim N \epsilon / L^2.$$

In the limit we are taking, $l_s^2 \sim N^{-1/2} \sim \epsilon \rightarrow 0$ and hence the $6d$ and $10d$ string scales, as expected, are going to zero in the same way. This could be used as an alternative way to argue for the choice made in (3.8). This gives a uniform picture for both of the near-BPS and far from BPS limits, that in both of the cases we scale the corresponding $6d$ string scale squared as ϵ to zero while keeping L fixed and that, certain massless fluctuations of these $6d$ strings are the degrees of freedom of the $2d$ dual CFT.

⁹Recalling the form of our $6d$ action (4.1) and that its vacuum solution is an AdS_6 of radius $\sqrt{2/5}L$ with $X = 1$, the tension of electrically and magnetically charged strings are equal.

which as expected is exactly the same number as the intersecting three-brane giants (*cf.* (2.8)). As discussed earlier, the intersecting giant graviton system is not supersymmetric even at the extremal point. In the reduced $6d$ gravity theory, however, we expect the system of N_2 electrically and N_3 magnetically charged strings to form a “dyonic” (Q_e, Q_m) -type string state. The mass (squared) of this state, as usual BPS dyonic states, is sum of the mass squares of electrically and magnetically charged strings (see (7.1)). Number of this dyonic strings is then the largest-common-divisor of N_2 and N_3 . As N_2 and N_3 are both scaling like N , the number of the dyonic strings bound states formed out of these strings is then expected to be scaling like N . These $6d$ dyonic (Q_e, Q_m) -strings is briefly discussed in the discussion section and more thorough analysis will be presented in [27].

As the final step in the entropy function prescription, the entropy of the black string system (and hence that of the $BTZ \times S^3$ solution) is given by the value of the entropy function (4.5) at its minimum, that is it is value at (4.8), which is

$$F(Q_e = \tilde{q}_2, Q_m = \tilde{q}_3) = \frac{\pi^3 \rho_0 \epsilon}{G_N^{(6)}} \cdot \frac{(v_1 v_2)^{\frac{3}{2}}}{v_1}. \tag{4.14}$$

Recalling (4.8) and that $R_{AdS}^2 = v_1$, $R_S^2 = v_2$, we see that $F(\tilde{q}_2, \tilde{q}_3) = S_{BH}$ given in (3.10) which is in turn equal to (3.15).

5. Perturbative addition of the third charge

So far we have discussed two near-horizon far from BPS decoupling limits of the two-charge $5d$ black hole type solutions. It is, however, possible to turn on the third charge. In this case, again there are two possibilities for an extremal solution, the BPS case for which $\mu = 0$, and the $\mu = \mu_c$ case where the function f in the metric (2.3) has a double horizon at $r = r_h \neq 0$ [17]. In the generic three-charge extremal but non-BPS case, unlike the two-charge case, the horizon radius is non-zero and as discussed in [22] taking the near-horizon limit leads to $AdS_2 \times S^3$ geometry with unequal AdS_2 and S^3 radii [22], see also [28].

On the other hand, one may ask whether it is possible to add the third charge as a “perturbation” to the two-charge system, that is adding the third charge q_1 such that in the near-horizon scaling $q_1 \ll q_2, q_3$. If this is possible then one expects to find a rotating BTZ black hole. In this section we show that indeed such a possibility can be realized for both of the near-BPS and far from BPS, but non-BPS decoupling limits, respectively discussed in sections 2.2.1 and 2.2.2. We also extend black hole entropy arguments of section 3 to these cases.

5.1 The near-horizon limit: the near-BPS case

In this section we extend the limit defined in (2.12), (2.13) by turning on the third charge q_1 in a perturbative manner, i.e.

$$q_1 = \epsilon^2 \hat{q}_1 \tag{5.1}$$

while keeping \hat{q}_1 fixed. The rest of parameters are scaled the same as before. Let us first consider the case corresponding to $\mu_1 \sim \mu_1^0 \neq 1$. In this case (*cf.* (2.3))

$$H_1 = 1 + \frac{\hat{q}_1}{\tilde{\rho}^2}, \quad a_1 = -\frac{1}{L} \sqrt{\hat{q}_1(\hat{\mu} + \hat{q}_1)} \frac{1}{\tilde{\rho}^2 + \hat{q}_1}, \quad (5.2a)$$

$$H_i = \epsilon^{-1} \frac{\hat{q}_i}{\tilde{\rho}^2} \quad i = 2, 3, \quad \Delta = \epsilon^{-2} \frac{\hat{q}_2 \hat{q}_3}{\tilde{\rho}^4} (\mu_1^0)^2 \quad (5.2b)$$

$$f = 1 - \frac{M}{\tilde{\rho}^2} + \frac{J^2}{4\tilde{\rho}^4} \quad (5.2c)$$

where M is defined in (2.13), $\hat{\mu}_c = \hat{q}_2 \hat{q}_3 / L^2$ and $J^2 = 4\hat{q}_1 \hat{q}_2 \hat{q}_3 / L^2 = 4\hat{\mu}_c \hat{q}_1$.

If we redefine $\hat{\rho}$ as [6]

$$\rho^2 = \tilde{\rho}^2 + \hat{q}_1 \quad (5.3)$$

the metric takes the form

$$ds^2/\epsilon = \mu_1^0 \left[-\frac{\rho^2 F(\rho)}{R_S^2} dt^2 + R_S^2 \frac{d\rho^2}{\rho^2 F(\rho)} + \frac{L^2}{R_S^2} \rho^2 (d\phi_1 + a_1 dt)^2 + R_S^2 d\Omega_3^2 \right] + \frac{L^2}{R_S^2} \frac{1}{\mu_1^0} \sum_{i=2,3} \hat{q}_i ((d\mu_i/\epsilon)^2 + (\mu_i^0)^2 d\psi_i^2), \quad (5.4)$$

where ψ_i are as defined in (2.13), $R_S^4 = \hat{q}_2 \hat{q}_3$ and

$$F(\rho) = 1 - \frac{M + 2\hat{q}_1}{\rho^2} + \frac{\hat{q}_1(\hat{\mu} + \hat{q}_1)}{\rho^4}. \quad (5.5)$$

In the above $\hat{\mu} = M + \hat{q}_2 \hat{q}_3 / L^2$. Note also that as discussed in section 2.2.1 in this case $d\mu_i/\epsilon = \text{fixed}$.

The first line in the metric (5.4) describes an $X_{M,J} \times S^3$ space where $X_{M,J}$ depending on the values of M^2 and J can be a rotating BTZ, conical space or global AdS_3 (see appendix B for more detailed discussion). Radius of the AdS_3 background (measured in units of $\sqrt{\epsilon}$) is

$$\ell^2 = \mu_1^0 R_S^2.$$

The mass M_{BTZ} and angular momentum J_{BTZ} (*cf.* appendix B) is then¹⁰

$$M_{\text{BTZ}} = \frac{M + 2\hat{q}_1}{\hat{\mu}_c} = \frac{\hat{\mu} + 2\hat{q}_1}{\hat{\mu}_c} - 1, \quad J_{\text{BTZ}} = 2\sqrt{\frac{\hat{q}_1(\hat{\mu} + \hat{q}_1)}{\hat{\mu}_c^2}}. \quad (5.6)$$

We should stress that the above metric is a rotating black hole only when the extremality bound $M_{\text{BTZ}} \geq J_{\text{BTZ}}$ holds (and also $\phi \in [0, 2\pi]$). In terms of the parameters we have in our solution, that is

$$M^2 \geq 4\hat{q}_1 \hat{q}_2 \hat{q}_3 / L^2. \quad (5.7)$$

¹⁰The physical angular momentum of the original $10d$ black-brane (or electric charge of the $5d$ black hole) corresponding to q_1 charge, J_1 , is related to J_{BTZ} as

$$J_1 = \frac{N^2 \epsilon^2}{4} \frac{\mu_c}{L^2} J_{\text{BTZ}}.$$

We will comment on this relation in section 6.2.1, in (6.23).

Note that M can be positive or negative. The above solution is a black hole at (Hawking) temperature (measured in units of $\sqrt{\epsilon\ell}$)

$$T_{\text{BTZ}} = \frac{\sqrt{M^2 - 4\hat{q}_1\hat{q}_2\hat{q}_3/L^4}}{2\pi\rho_h\hat{\mu}_c}, \quad \rho_h^2 = \frac{1}{2\hat{\mu}_c} \left(M + 2\hat{q}_1 + \sqrt{M^2 - 4\hat{q}_1\hat{q}_2\hat{q}_3/L^4} \right). \quad (5.8)$$

For the special case of $M^2 = 4\hat{q}_1\hat{q}_2\hat{q}_3/L^2$ we have an extremal rotating BTZ which has $T_{\text{BTZ}} = 0$.

When $M_{\text{BTZ}} \leq -J_{\text{BTZ}} \leq 0$, we will have a sensible conical singularity (see appendix B) only if (5.7) holds while $M + 2\hat{q}_1 \leq 0$, that is

$$M \leq -2 \text{Max}(\hat{q}_1, \sqrt{\hat{q}_1\hat{q}_2\hat{q}_3/L^2}), \quad (5.9)$$

and if γ , $\gamma^2 \equiv J_{\text{BTZ}} - M_{\text{BTZ}}$, is a rational number.

In sum, in order to have a sensible string theory description we should have

$$M_{\text{BTZ}} - J_{\text{BTZ}} + 1 \geq 0, \quad (5.10)$$

and if $0 \leq J_{\text{BTZ}} - M_{\text{BTZ}} \equiv \gamma^2 \leq 1$, γ should be rational.

In a similar manner one can extend (2.12) to the case with non-zero \hat{q}_1 . The result is the same as (2.17) but the AdS_3 part is again replaced with a rotating BTZ.

It is straightforward to generalize the discussions of section 3 to this case and compute the Bekenstein-Hawking entropy of the above $5d$ black hole solutions after taking the limit¹¹

$$S_{\text{BH}} = \pi N^2 \epsilon^2 \frac{\hat{\mu}_c}{L^2} \rho_h. \quad (5.11)$$

As we see the entropy, similarly to the static case of section 3.1, scales as $N^2\epsilon^2$. The curvature radii square, however, scale as $\epsilon\ell^2$. In order for the gravity description to be valid, one needs to ensure $\epsilon\ell^2 \gg \alpha'$ or $\epsilon\sqrt{N} \gg \ell^2/L^2$, which is fulfilled by taking L to be parameterically larger than ℓ , and scaling $l_s^2 \sim \epsilon$ or equivalently $N \sim \epsilon^{-2}$. In this case, the entropy scales as $N \sim \epsilon^{-2} \rightarrow \infty$.

5.2 The near-horizon limit: the far from BPS case

In this part we extend the limit discussed in section 2.2.2 in equation (2.19) by turning on the third charge q_1 “perturbatively”, with the scaling

$$q_1 = \epsilon^4 \hat{q}_1. \quad (5.12)$$

¹¹Note that the expression for ρ_h^2 is nothing but

$$\rho_h^2 = \frac{1}{2} \left(M_{\text{BTZ}} + \sqrt{M_{\text{BTZ}}^2 - J_{\text{BTZ}}^2} \right)$$

yielding

$$\rho_h = \frac{1}{2} \left(\sqrt{M_{\text{BTZ}} - J_{\text{BTZ}}} + \sqrt{M_{\text{BTZ}} + J_{\text{BTZ}}} \right).$$

Therefore, we have a matching between the entropy of the $5d$ blackhole and that of $3d$ rotating BTZ.

In this case, $H_1 \simeq 1$ and Δ is given in (2.20) but now f is

$$f = f_0 - \frac{M}{\tilde{\rho}^2} + \frac{J^2}{4\tilde{\rho}^4}, \quad J^2 \equiv 4 \frac{\hat{q}_1 q_2 q_3}{L^2} = 4\mu_c \hat{q}_1 \quad (5.13)$$

where $f_0 = 1 + \frac{q_2+q_3}{L^2}$. After taking the above limit the metric takes the form

$$ds^2 = \mu_1 [R_{\text{AdS}}^2 ds_{\text{BTZ}}^2 + R_S^2 d\Omega_3^2] + \frac{1}{\mu_1} d\mathcal{M}_4^2 \quad (5.14)$$

where $d\mathcal{M}_4^2$ is given in (2.23), $R_S^4 = q_2 q_3$, $R_{\text{AdS}}^2 = R_S^2/f_0$ and

$$ds_{\text{BTZ}}^2 = -N(\rho)d\tau^2 + \frac{d\rho^2}{N(\rho)} + \rho^2(d\varphi - N_\varphi d\tau)^2 \quad (5.15)$$

in which

$$N(\rho) = \rho^2 - M_{\text{BTZ}} + \frac{J_{\text{BTZ}}^2}{4\rho^2}, \quad N_\varphi = \frac{J_{\text{BTZ}}}{2\rho^2}, \quad (5.16)$$

with

$$M_{\text{BTZ}} = \frac{M}{\mu_c}, \quad J_{\text{BTZ}} = 2\sqrt{\frac{f_0 \hat{q}_1}{\mu_c}}. \quad (5.17)$$

Note that although the metric (5.15) has the standard form of a rotating BTZ black hole [29] (see also appendix B) we should stress that the range of the angular coordinate φ is $[0, 2\pi\epsilon]$. The new time coordinates τ, ρ are related to the original $10d$ coordinates as in (2.25). The above geometry has the interpretation of rotating BTZ only if $N(\rho) = 0$ has real solutions, that is when

$$M^2 \geq 4\mu_c f_0 \hat{q}_1. \quad (5.18)$$

The Bekenstein-Hawking entropy of the above rotating BTZ is

$$S_{\text{BH}} = \pi \rho_h \frac{1}{\sqrt{f_0}} \frac{\mu_c}{L^2} N^2 \epsilon, \quad \rho_h = \frac{1}{2} \left(\sqrt{M_{\text{BTZ}} + J_{\text{BTZ}}} + \sqrt{M_{\text{BTZ}} - J_{\text{BTZ}}} \right), \quad (5.19)$$

which turns out to be exactly the expression one would obtain after taking the decoupling limit on the Bekenstein-Hawking entropy of the corresponding three-charge $5d$ black hole.

It is also immediate to see that repeating the entropy function analysis of section 4 for the rotating BTZ case, we will exactly obtain the entropy for the system of corresponding $6d$ black strings. The $6d$ analysis for this case is again suggestive of existence of $6d$ string picture. We will comment further on this point in section 6.2.2.

6. The dual gauge theory descriptions

So far we have shown that one can take specific near-horizon, near-extremal limits over $10d$ type IIB solutions which are asymptotically AdS_5 . As such one would expect that these solutions, the limiting procedure and the resulting geometry after the limit should have a dual description via AdS_5/CFT_4 . On the other hand, after the limit we obtain a space which contains $AdS_3 \times S^3$ and hence there should also be another dual description in terms of a $2d$ CFT. In this section we intend to study both $4d$ and $2d$ dual CFT descriptions.

6.1 Description in terms of $d = 4, \mathcal{N} = 4$ SYM

In this section we first identify the operators of $\mathcal{N} = 4, d = 4$ $U(N)$ SYM theory, by specifying their $SO(4, 2) \times SO(6)$ quantum numbers, corresponding to the two-charge gravity solutions discussed in earlier sections. We then translate taking the near-horizon, near-extremal limits in the dual $\mathcal{N} = 4$ theory and identify the sector of the gauge theory operators corresponding to string theory on the decoupled backgrounds for both of the near-BPS and far from BPS cases.

According to the standard AdS/CFT dictionary [2], the scaling dimension of operators Δ and their R -charge J_i respectively correspond to the ADM mass and angular momentum of the objects in the gravity side. Explicitly, for the two-charge case of our interest, the operators are specified by three quantum numbers [6, 1, 26]:¹²

$$\begin{aligned} \Delta &= L \cdot M_{\text{ADM}} = \frac{N^2}{2L^2} \left(\frac{3}{2} \mu + q_2 + q_3 \right), \\ J_i &= \frac{\pi L}{4G_5} \tilde{q}_i = \frac{N^2}{2} \frac{\tilde{q}_i}{L^2}. \end{aligned} \tag{6.1}$$

where M_{ADM} has the same expression as M (2.5b) but the last term, the Casimir energy, has been dropped. Operators in this sector are singlets of the $SO(4) \in SO(4, 2)$. As we see in both cases, if μ and q_i are finite, Δ and J_i both scale like N^2 .

In both of the near-BPS and far from BPS limits we are taking the 't Hooft coupling, $\lambda = L^4/l_s^4$ to infinity and one should not expect the dual $4d$ field theory to give a useful, i.e. a perturbative, description. On the other hand, after BMN [15], we have learned that the effective expansion parameters of the $4d$ gauge theory may be different in sectors of large R -charges such that for specific “almost-BPS” operators the effective (or “dressed”) 't Hooft coupling and the genus expansion parameter remains finite. In this subsection we try to extend the ideology of BMN to the new “almost-BPS” as well as “almost-extremal” sectors.

6.1.1 Near-horizon near-BPS limit, $\mathcal{N} = 4$ SYM description

In the near-BPS limit case together with some of the coordinates we also scale μ and q_i as ϵ . As discussed in section 3.1.1 (see also footnote 8) we need to also scale $N \sim \epsilon^{-2}$. Therefore, in this limit Δ and J_i of the operators scale as:

$$\begin{aligned} \Delta &= \frac{N^2 \epsilon}{2} (\hat{q}_2 + \hat{q}_3 + \mathcal{O}(\epsilon))/L^2 \sim N^{3/2} \rightarrow \infty \\ J_i &= \frac{N^2 \epsilon}{2} (\hat{q}_i + \mathcal{O}(\epsilon))/L^2 \sim N^{3/2}. \end{aligned} \tag{6.2}$$

That is, the sector of the $\mathcal{N} = 4$ $U(N)$ SYM operators corresponding to the geometries in question have large scaling dimension and R -charge, $\Delta \sim J_i \sim N^{3/2}$. In the same spirit as the BMN limit [15], one can find certain combinations of Δ and J_i which are finite and describe physics of the operators after the limit. To find these combinations we recall the

¹²In our conventions the BPS condition in the gauge theory side is written as $\Delta = \sum_i J_i$.

way the limit was taken, i.e. (2.12),¹³ and in particular note that¹⁴

$$iL \frac{\partial}{\partial \tau} = iL \frac{\partial}{\partial t} + i \sum_{i=2,3} \frac{\partial}{\partial \phi_i} = \Delta - \sum_{i=2,3} J_i \quad (6.3a)$$

$$-i \frac{\partial}{\partial \psi_i} = -i \frac{\partial}{\partial \phi_i} = J_i \quad (6.3b)$$

Up to leading order we have

$$\begin{aligned} \Delta - \sum_{i=2,3} J_i &= \frac{N^2 \epsilon^2}{4} \frac{\hat{\mu}}{L^2}, \\ J_i &= \frac{N^2 \epsilon}{2} \frac{\hat{q}_i}{L^2}. \end{aligned} \quad (6.4)$$

As we see $\Delta - \sum J_i$ scales as $N^2 \cdot N^{-1} = N \rightarrow \infty$, while $J_i \sim N^{3/2}$ and therefore the ‘‘BPS deviation parameter’’ [31]

$$\eta_i \equiv \frac{\Delta - \sum_i J_i}{J_i} \sim \epsilon \sim N^{-1/2} \rightarrow 0, \quad (6.5)$$

and hence we are dealing with an ‘‘almost-BPS’’ sector.¹⁵ Moreover, $\Delta - \sum J_i$ is linearly proportional to non-extremality parameter $\hat{\mu}$. It is also notable that S_{BH} (3.7) scales the same as $\Delta - \sum J_i$.

To write (6.4) in terms of the gauge theory parameters we need to replace ϵ for parameters of the gauge theory, which we choose

$$\epsilon = \frac{2}{\sqrt{N}}. \quad (6.6)$$

In sum, the sector we are dealing with is composed of ‘‘almost (at most) 1/4 BPS’’ operators of $U(N)$ SYM with¹⁶

$$\begin{aligned} \Delta \sim J_i \sim N^{3/2}, & \quad \lambda = g_{\text{YM}}^2 N \sim N \rightarrow \infty \\ \frac{J_i}{N^{3/2}} \equiv \frac{\hat{q}_i}{L^2} = \text{fixed}, & \quad \left(\Delta - \sum_{i=2,3} J_i \right) \cdot \frac{1}{N} = \frac{\hat{\mu}}{L^2} = \text{fixed}. \end{aligned} \quad (6.7)$$

¹³In the near-BPS case we discussed two limits, $\mu_1 \simeq 1$ and $\mu_1 \neq 1$ of (2.13). For the latter one also scales ψ_i with respect to ϕ_i . As a result, (6.3a) remains unchanged while (6.3b) is modified to $-i \frac{\partial}{\partial \psi_i} = \epsilon^{1/2} J_i$. In this section, however, we are only going to utilize (6.3a).

¹⁴In the Penrose-BMN limit, *à la* Tseytlin, the $AdS_5 \times S^5$ coordinates τ, ψ are related to the ‘‘light-cone’’ coordinates x^+, x^- as [30]

$$t = x^+, \quad \psi = x^+ - \frac{1}{R^2} x^-$$

where R is the AdS radius which is taken to infinity as $N^{1/4}$.

¹⁵It is instructive to make parallels with the BMN sector [15]. In the BMN sector we are dealing with operators with

$$\Delta \sim J \sim N^{1/2}, \quad \text{while} \quad \Delta - J = \text{finite},$$

implying that, similarly to our case, $\eta_{\text{BMN}} \sim N^{-1/2} \rightarrow 0$.

¹⁶Note that the value for $\Delta - \sum J_i$ read from the gravity side, via AdS/CFT, corresponds to the strong 't Hooft coupling regime of the gauge theory.

The dimensionless physical quantities that describe this sector are therefore \hat{q}_i/L^2 , $\hat{\mu}/L^2$ and g_{YM} .

To specify the sector completely we should also determine the *basis* we use to contract the $N \times N$ $U(N)$ gauge indices. This could be done by giving the (approximate) shape of the corresponding Young tableaux. To this end we recall the interpretation of the original $10d$ geometry in terms of the back-reaction of the intersecting giant gravitons and that giant gravitons and their open string fluctuations are described by (sub)determinant operators [9, 12, 13, 31]. Here we are dealing with a system of intersecting multi giants. The “number of giants” in each stack in the near-BPS, near-horizon limit is (2.8)

$$N_i = N\epsilon \cdot \frac{\hat{q}_i}{L^2} = 2N^{1/2} \frac{\hat{q}_i}{L^2}, \quad (6.8)$$

and therefore, $\Delta - \sum_i J_i = \frac{N_2 N_3}{4} \frac{\hat{\mu}}{\hat{\mu}_c}$.

Finally, let us consider the rotating case of section 5.1, where besides J_2 , J_3 we have also turned on the third R -charge J_1 ,

$$J_1 = \frac{N^2 \epsilon^2}{2} \cdot \frac{1}{L^2} \sqrt{\hat{q}_1(\hat{q}_1 + \hat{\mu})}. \quad (6.9)$$

As we see $\Delta - \sum_{i=2,3} J_i \sim J_1 \sim N^2 \epsilon^2 \sim N \rightarrow \infty$. (Note that in this case, as we have also turned on the third charge q_1 , Δ is not the expression given in (6.1) and one should use (2.5).) Instead of $\Delta - \sum_{i=2,3} J_i$ it is more appropriate to define another positive definite quantity:

$$\Delta - \sum_{i=1}^3 J_i = N \cdot \left(\frac{\hat{\mu} + 2\hat{q}_1 - \sqrt{(\hat{\mu} + 2\hat{q}_1)^2 - \hat{\mu}^2}}{L^2} \right) \geq 0. \quad (6.10)$$

It is remarkable that the above BPS bound is exactly the same as the bound (5.10). This bound is more general than just the extremality bound of the rotating BTZ black hole $M_{\text{BTZ}} \geq J_{\text{BTZ}} \geq 0$. This bound besides the rotating black hole cases also includes the case in which we have a conical singularity which could be resolved in string theory (*cf.* appendix B and section 5.1). We will comment on this point further in section 6.2.1.

6.1.2 Near-horizon far from BPS limit, $\mathcal{N} = 4$ SYM description

Since in the near-horizon, far from BPS limit of (2.19) we do not scale μ and q_i 's, in this case we expect to deal with a sector of $\mathcal{N} = 4$ SYM in which $\Delta \sim J_i \sim N^2$ and, as discussed in 3.1.2, $N \sim \epsilon^{-1}$. To deduce the correct “BMN-type” combination of Δ and J_i which correspond to physical observables, we again recall the way the limit has been taken, and in particular

$$\tau = \epsilon \frac{R_S}{R_{\text{AdS}_3}} \frac{t}{L}, \quad \phi_i = \psi_i + \frac{\tilde{q}_i R_{\text{AdS}_3}}{q_i R_S} \frac{\tau}{\epsilon}, \quad i = 2, 3. \quad (6.11)$$

Therefore, $-i \frac{\partial}{\partial \psi_i} = -i \frac{\partial}{\partial \phi_i} = J_i$ and

$$\mathcal{E} \equiv -i \frac{\partial}{\partial \tau} = -\frac{R_{\text{AdS}_3}}{\epsilon R_S} \left(iL \frac{\partial}{\partial t} + i \sum_{i=2,3} \frac{\tilde{q}_i}{q_i} \frac{\partial}{\partial \phi_i} \right) = -\frac{R_{\text{AdS}_3}}{\epsilon R_S} \left(\Delta - \frac{2L^2}{N^2} \sum_{i=2,3} \frac{J_i^2}{q_i} \right) \quad (6.12)$$

The last equality can be understood in an intuitive way. In the near-extremal case we are indeed dealing with massive giant gravitons which are far from being BPS and hence are behaving like *non-relativistic* objects which are rotating with angular momentum J_i over circles with radii R_i , $R_i^2 = \frac{L^2}{R_S^2} q_i$ (2.23). Therefore, the kinetic energy of this rotating branes is proportional to $\sum J_i^2/q_i$. As discussed in section 3.2 in our limit $\epsilon \sim 1/N$ which for convenience we choose

$$\epsilon = \frac{4}{N}. \tag{6.13}$$

Recalling that Δ is measuring the “total” energy of the system, then \mathcal{E} should have two parts: the rest mass of the system of giants and the energy corresponding to the “internal” excitations of the branes. To see this explicitly we recall (6.1), (2.5) and (4.11), yielding

$$\begin{aligned} \mathcal{E} &= \frac{R_{\text{AdS}_3}}{R_S} \cdot \frac{N^2}{4\epsilon} \cdot \frac{\mu}{L^2} \\ &= \mathcal{E}_0 + \frac{R_{\text{AdS}_3}}{R_S} \cdot (2\pi T_s^{(6)} M) \end{aligned} \tag{6.14}$$

where we have used $\mu = \mu_c + \epsilon^2 M$ (M is related to the mass of BTZ black hole (2.24)), and

$$\mathcal{E}_0 = \frac{R_{\text{AdS}_3} R_S^3}{16L^4} \cdot N^3. \tag{6.15}$$

\mathcal{E}_0 which is basically \mathcal{E} evaluated at $\mu = \mu_c$, is the rest mass of the brane system.¹⁷

$\mathcal{E} - \mathcal{E}_0$ which is proportional to $T_s^{(6)} M$ corresponds to the fluctuations of the giants about the extremal point. The fact that $\mathcal{E} - \mathcal{E}_0$ is proportional to $T_s^{(6)} M$ indicates that it can be recognized as a fluctuations of a $6d$ string. Recall also that from the $10d$ viewpoint, the $6d$ strings are uplifted to three-brane giants with two legs along the \mathcal{M}_4 directions. Therefore, $\mathcal{E} - \mathcal{E}_0$ corresponds to (three) brane-type fluctuations of the original “intersecting giants”.

In sum, from the $U(N)$ SYM theory viewpoint the sector describing the near-horizon far from BPS limit consists of operators specified with

$$\begin{aligned} \Delta &\sim J_i \sim N^2, & \lambda &\sim N \rightarrow \infty, \\ \frac{J_i}{N^2} &\equiv \frac{\tilde{q}_i}{2L^2} = \text{fixed}, & \frac{\mathcal{E} - \mathcal{E}_0}{N} &= \text{fixed}, \end{aligned} \tag{6.16}$$

¹⁷One should keep in mind that at the extremal point the system is not BPS and hence the “rest mass” of the giants system is not simply sum of the masses of individual stacks of giants and already contains their “binding energy” (stored in the deformation of the giant shape from the spherical shape). Nonetheless, it should still be proportional to the number of giants times mass of a single giant. In the $6d$ language, as suggested in section 4, this corresponds to formation of a $6d$ (Q_e, Q_m) -string. eq.(6.15), however, seems to suggest a simpler interpretation in terms of dual giants [8]. Inspired by the expression for the $10d$ five-form flux and recalling that the IIB five-form is self-dual, the system of giants we start with, e.g. through SUGRA solution (2.1), may also be interpreted as spherical three-branes wrapping $S^3 \in AdS_5$ while rotating on S^5 . After the limit, we are dealing with a system of dual giants wrapping the $S^3 \in AdS_3 \times S^3$, of radius R_S . The mass of a single such dual giant m_0 (as measured in R_{AdS_3} units and also noting the scaling of AdS_5 time with respect to AdS_3 time) is then

$$\frac{m_0}{R_{\text{AdS}_3}/\epsilon} = T_3(2\pi^2 R_S^3) = \frac{R_S^3}{L^4} \cdot N.$$

The number of dual giants is again proportional to N and hence one expects the total “rest mass” of the system m_0 to be proportional to $N^3 R_S^3$.

where \mathcal{E} , \mathcal{E}_0 in equations (6.12), (6.14) and (6.15) are defined in terms of Δ , J_i .

As discussed in section 5.2 one may obtain a rotating BTZ if we turn on the third R -charge in a perturbative manner. In the $4d$ gauge theory language this is considering the operators which besides being in the sector specified by (6.16) carry the third R -charge J_1 , $J_1 \sim N^2 \epsilon^2 \sim 1$. Explicitly,

$$J_1 = \frac{N^2}{2L^2} \epsilon^2 \sqrt{\hat{q}_1 \mu_c} \tag{6.17}$$

One should, however, note that in terms of the AdS_3 parameters, since $\varphi = \epsilon \phi$, then

$$\mathcal{J} \equiv -i \frac{\partial}{\partial \varphi} = -i \frac{1}{\epsilon} \frac{\partial}{\partial \phi} = \frac{J_1}{\epsilon} = \frac{N^2 \epsilon}{2} \frac{\mu_c}{L^2} \sqrt{\frac{\hat{q}_1}{\mu_c}} \tag{6.18}$$

As we see \mathcal{J} , similarly to $\mathcal{E} - \mathcal{E}_0$, is also scaling like $N^2 \epsilon \sim N$ in our decoupling limit. When J_1 is turned on the expressions for the Δ and hence \mathcal{E} are modified, receiving contributions from q_1 . These corrections, recalling (2.5) and that q_1 scales as ϵ^4 (5.12), vanish in the leading order. However, one may still define physically interesting combinations like $\mathcal{E} - \mathcal{E}_0 \pm \mathcal{J}$. We will elaborate further on this point in section 6.2.2.

Before closing this subsection some comments are in order:

- The remarkable point which is seen directly from (6.12) is that $-\mathcal{E}$ is negative definite, i.e. there is an *extremality bound*:

$$\Delta - \sum_i f_i(J_i) \leq 0. \tag{6.19}$$

where

$$f_i(J_i) = \frac{2L^2}{N^2} \frac{J_i^2}{q_i}.$$

(Note that one can express q_i in terms of the J_i 's but since the explicit expressions are not illuminating we do not present them here.) This could be thought of as a complement to the usual BPS bound, $\Delta - \sum_i J_i \geq 0$.

- We note that both $\mathcal{E} - \mathcal{E}_0$ and \mathcal{J} scale as $N^2 \epsilon \sim N$ which is the same scaling as the black hole entropy (3.10).
- Finally, the system of original intersecting giants is composed of two stacks of D3 giants each containing $N_i = N \frac{\hat{q}_i}{L^2}$ branes and $N_i \sim N \rightarrow \infty$.

6.2 Description in terms of the dual 2d theory

As we showed in either of the near-BPS or far from BPS near-horizon limits we obtain a spacetime which has an $AdS_3 \times S^3$ factor. This, within the AdS/CFT ideology, is suggesting that (type IIB) string theory on the corresponding geometries should have a dual 2d CFT description. In this section we elaborate on this 2d description.

6.2.1 Near-BPS case, the dual 2d CFT description

This case was discussed in [1] and references therein and hence we will be brief about it. The metric in this case takes the same form as the near-horizon limit of a D1-D5 system, though the AdS_3 is obtained to be in *global* coordinates. This could be understood noting that the geometry 2.1, in the two-charge case, corresponds to a system of *smear*ed giant D3-branes intersecting on a circle. In the near-horizon limit, however, we take the radius of the giants to be very large (or equivalently focus on a very small region on the worldvolume of the spherical brane) while keeping the radius of the intersection circle to be finite (in string units). Therefore, upon two T-dualities on the D3-branes along the \mathcal{C}_4 directions the system goes over to a D1-D5 system but now the D1 and D5 are lying on the circle (D5 has its other four directions along \mathcal{C}_4). The situation is essentially the same as the usual D1-D5 system, e.g. see [3, 32] for reviews, with only an important difference [1]. Here we just give the dictionary from our conventions and notations of the usual D1-D5 system (for a detailed review see [32]) and those of [23, 33], and discuss the difference.

- Number of D-strings Q_1 and number of D5 Q_5 are respectively equal to the number of giants in each stack N_2 and N_3 (6.8).
- The degrees of freedom are coming from four DN modes of open strings stretched between intersecting giants which are in (N_2, \bar{N}_3) representation of $U(N_2) \times U(N_3)$.¹⁸
- In taking the near-horizon, near-BPS limit we are focusing on a narrow strip on μ_2, μ_3 directions and hence our $BTZ \times S^3 \times \mathcal{C}_4$ geometry and in this sense the corresponding $\mathcal{N} = (4, 4)$ 2d CFT description is only describing the narrow strips on the original 5d black hole. Therefore, our 5d black hole is described in terms of not a single 2d CFT, but a collection of (infinitely many of) them. The *only* property which is different among these 2d CFT's is their central charge. The “metric” on the space of these 2d CFT's is exactly the same as the metric on \mathcal{C}_4 . Therefore, as far as the entropy and the overall (total) number of degrees of freedom are concerned, one can define an *effective central charge* of the theory which is the integral over the central charge of the theory corresponding to each strip [1]. To compute the central charge we use the Brown-Henneaux central charge formula [35],

$$c = \frac{3 R_{\text{AdS}}}{2 G^{(3)}}$$

and recall that in our case for each strip $R_{\text{AdS}}^2 = \mu_1^0 R_S^2$, that $G^{(3)} \propto \sqrt{\mu_1^0 / \mu_2^0 \mu_3^0}$ and *effective* total central charge is obtained by integrating strip-wise c over the \mathcal{C}_4 . Noting that the central charge of the usual D1-D5 system is given by $6Q_1Q_5$, and that

$$\int_{\mu_2^2 + \mu_3^2 \leq 1} \mu_2 \mu_3 d\mu_2 d\mu_3 = \frac{1}{8},$$

¹⁸It has been shown, from DBI action analysis [34] and using the description of giants in the $\mathcal{N} = 4$ SYM in [11], that similarly to flat D-brane case, when we have a N number of giants sitting on top of each other the low energy effective field theory becomes a $U(N)$ gauge theory on the giant.

the *effective* central charge of the system is

$$c_L = c_R = c = 3N_2N_3 = 12N \cdot \frac{\hat{\mu}_c}{L^2}. \quad (6.20)$$

It is notable that in our case the central charge $c \sim N \rightarrow \infty$.

- In contrast to the results of [1] e.g. eq. (2.22) or eq. (4.79) there, we should stress that in our case the entropy, and hence the central charge c are scaling like N , as opposed to N^2 there. This difference appears recalling that in our case the entropy scales as $N^2\epsilon^2$ and that $\epsilon^2 \sim 1/N$.

- As discussed in the appendix B the generic solution can be a rotating BTZ black hole or conical singularity, if

$$M - J \geq -1.$$

- The 2d CFT is described by L_0, \bar{L}_0 (respectively equal to the left and right excitation number of the 2d CFT N_L and N_R , divided by N_2N_3) which are related to the BTZ black hole mass and angular momentum [32] as

$$L_0 = \frac{6}{c}N_L = \frac{1}{4}(M_{\text{BTZ}} - J_{\text{BTZ}}), \quad \bar{L}_0 = \frac{6}{c}N_R = \frac{1}{4}(M_{\text{BTZ}} + J_{\text{BTZ}}). \quad (6.21)$$

The above expressions for L_0, \bar{L}_0 are given for $M_{\text{BTZ}} - J_{\text{BTZ}} \geq 0$ when we have a black hole description. When $-1 \leq M_{\text{BTZ}} - J_{\text{BTZ}} < 0$, we need to replace them with $L_0 = -\frac{c}{24}a_+^2$, $\bar{L}_0 = -\frac{c}{24}a_-^2$ (in the conventions introduced in the appendix B) [32, 36]. In the special case of *global* AdS_3 background, where $a_+ = a_- = 1/2$ formally corresponding to $M_{\text{BTZ}} = -1, J_{\text{BTZ}} = 0$, the ground state is describing an NSNS vacuum of the $\mathcal{N} = (4, 4)$ 2d CFT [33, 23].¹⁹ The expressions for M_{BTZ} and J_{BTZ} in terms of the system of giants are given in (5.6).

- With the above identification, it is readily seen that the Cardy formula for the entropy of a 2d CFT

$$\begin{aligned} S_{2d \text{ CFT}} &= 2\pi \left(\sqrt{cN_L/6} + \sqrt{cN_R/6} \right) \\ &= \frac{\pi}{6} c \left(\sqrt{M_{\text{BTZ}} - J_{\text{BTZ}}} + \sqrt{M_{\text{BTZ}} + J_{\text{BTZ}}} \right) \end{aligned} \quad (6.22)$$

exactly reproduces the expressions for the entropy we got in the previous section, (5.11), once we substitute for the central charge from (6.20) and $M_{\text{BTZ}}, J_{\text{BTZ}}$ from (5.6).

- Although the entropy and the energy of the system (which are both proportional to the central charge) grow like N and go to infinity in the limit we are interested in, the temperature and the horizon size (5.8) remain finite.

¹⁹In *global* AdS_p spaces, when p is odd the expression for the ADM mass has a Casimir energy [19]; for AdS_3 , in units of AdS radius (R), the Casimir energy is given by $R/8G_N^{(3)}$.

- It is also instructive to directly compare the $4d$ description discussed in 6.1.1 and the $2d$ field theory descriptions. Comparing the expressions for M_{BTZ} , J_{BTZ} and $\Delta - \sum_{i=2,3} J_i$, J_1 , we see that they match; explicitly

$$\Delta - \sum_{i=2,3} J_i = \frac{c}{12}(M_{\text{BTZ}} + 1), \quad J_1 = \frac{c}{12}J_{\text{BTZ}}. \quad (6.23)$$

This is very remarkable because it makes a direct contact between the $2d$ and $4d$ gauge theory descriptions. The $4d$ gauge theory BPS bound, i.e. $\Delta - \sum_{i=1,2,3} J_i \geq 0$ now translates into the bound $M_{\text{BTZ}} - J_{\text{BTZ}} \geq -1$. This means that the $4d$ gauge theory, besides being able to describe the rotating BTZ black holes, can describe the conical spaces too. In other words, $\Delta - \sum_{i=1}^3 J_i = 0$ and $N \frac{\hat{\mu}_c}{L^2}$ respectively correspond to global AdS_3 and massless BTZ cases and when

$$0 < \Delta - \sum_{i=1}^3 J_i < \frac{c}{12} = N \frac{\hat{\mu}_c}{L^2},$$

the $4d$ gauge theory is describing a conical space, provided that γ ,

$$\gamma^2 \equiv \frac{12}{c} \left(\Delta - \sum_{i=1}^3 J_i \right) - 1,$$

is a rational number. This is of course expected if the dual gauge theory description is indeed describing string theory on the conical space background. One should also keep in mind that entropy and temperature are sensible only when $\Delta - \sum_{i=1}^3 J_i \geq \frac{c}{12}$; for smaller values the degeneracy of the operators in the $4d$ gauge theory is not large enough to form a horizon of finite size (in $3d$ Planck units).

6.2.2 Far from BPS case, the dual $2d$ CFT description

As discussed before, in the near-horizon limit over a near-extremal two-charge black hole we again obtain an $AdS_3 \times S^3$ in which the AdS_3 and S^3 factors have different radii, moreover, although locally AdS_3 , the coordinate parameterizing $S^1 \in AdS_3$ is ranging over $[0, 2\pi\epsilon] = [0, 8\pi/N]$. As such, one expects the dual $2d$ CFT description to have somewhat different properties than the standard D1-D5 system. Based on the analysis and results of previous sections we conjecture that there exists a $2d$ CFT which describes the $6d$ string theory on this $AdS_3 \times S^3$ geometry. This string theory could be embedded in the $10d$ IIB string theory on the background (5.14).

Here we just make some remarks about this conjectured $2d$ CFT and a full identification and analysis of this theory is postponed for future works:

- This $2d$ CFT resides on the $R \times S^1$ causal boundary of the $AdS_3 \times S^3$ geometry (*cf.* discussions of section 2.2.2).²⁰

²⁰It is worth noting that in terms of the coordinates t and ϕ_1 of the original AdS_5 background, as noted in (2.27) we have a space which looks like a (supersymmetric) orbifold of AdS_3 [37], by $Z_{\epsilon-1}$, that is an $AdS_3/Z_{N/4}$. It is desirable to understand our analysis from this orbifold viewpoint.

- Noting (2.27), one may use the Brown-Henneaux analysis [35] to compute the central charge of this $2d$ CFT

$$c = \frac{3R_{\text{AdS}_3}\epsilon}{2G_N^{(3)}} = 12 \frac{\mu_c}{L^2 \sqrt{f_0}} N. \quad (6.24)$$

As we see in this case the expression for the central charge, except for the $1/\sqrt{f_0}$ factor, is the same as that of the near-BPS case (6.20), and scales like $N \rightarrow \infty$ in our limit.

- The $5d$ or $3d$ black hole entropies given in (5.19) take exactly the same form obtained from counting the number of microstates of a $2d$ CFT, i.e. the Cardy formula (6.22), with the central charge (6.24) and M_{BTZ} and J_{BTZ} given in (5.17).
- As discussed in section 6.1.2, there is a sector of $\mathcal{N} = 4$, $d = 4$ SYM which describes the IIB string theory on the background (5.14). This sector is characterized by $\mathcal{E} - \mathcal{E}_0$ and \mathcal{J} . From (6.14) and (6.18) one can readily express the $4d$ parameters in terms of $2d$ parameters, namely:

$$\mathcal{E} - \mathcal{E}_0 = \frac{c}{12} M_{\text{BTZ}}, \quad \mathcal{J} = \frac{c}{12} J_{\text{BTZ}}, \quad (6.25)$$

where c is given in (6.24) and M_{BTZ} , J_{BTZ} are given in (5.17). The above relations have of course the standard form of the usual D1-D5 system, and/or the near-BPS case discussed in section 6.2.1. Note, however, that in this case $\mathcal{E} - \mathcal{E}_0$ is measuring the mass of the BTZ with the zero point energy set at the massless BTZ case (rather than global AdS_3).

- As discussed in sections 4 and 6.1.2 we expect the degrees of freedom of this $2d$ CFT to correspond to $10d$ IIB string states on the $AdS_3 \times S^3$ geometry discussed in (4.1), which in turn correspond to brane-like excitations about the extremal intersecting giant three-branes. It is of course desirable to make this picture precise and explicitly identify the corresponding $2d$ CFT.

7. Summary and discussion

In this paper we extended the analysis of [1] and discussed in more details the near-horizon decoupling limits of the near-extremal two-charge black holes of $U(1)^3$ $d = 5$ gauged SUGRA. We showed that there are *two* such decoupling limits, one corresponding to *near-BPS* and the other to *far from BPS* black hole solutions. There were similarities and differences between the two cases. In both cases taking the limit over the uplift of the $5d$ black hole solution to $10d$ IIB theory, we obtain a geometry containing an $AdS_3 \times S^3$ factor (or more generally a $X_{M,J} \times S^3$ geometry, where $X_{M,J}$ is generically a $3d$ stationary spacetime the Ricci curvature of which obeying $R_{\mu\nu} = -\frac{1}{R^2}g_{\mu\nu}$). Therefore, there should be a $2d$ CFT dual description. On the other hand, noting that the starting $5d$ (or $10d$) geometry is a solution in the AdS_5 (or $AdS_5 \times S^5$) background there is a description in terms of the dual $4d$ SYM theory. We identified the central charge of the dual $2d$ CFT's

in both cases and showed that the Bekenstein-Hawking entropy of the original $5d$ solution, which is the same as the Bekenstein-Hawking entropy of the $3d$ BTZ black hole obtained after the limit, is correctly reproduced by the Cardy formula of a $2d$ CFT, from which we identified the L_0, \bar{L}_0 of the corresponding $2d$ CFT's in terms of the parameters of the original $5d$ black hole. Matching of the Bekenstein-Hawking entropy of the $5d$ and $3d$ black holes is a strong indication that the near-horizon limit we are taking is indeed a “decoupling” limit.

For the near-BPS case, the $2d$ description is essentially the same as that of the D1-D5 system, modulo the complication that our background corresponds not to a single $\mathcal{N} = (4, 4)$ $2d$ CFT but a (continuous) collection of them, all of which have the same L_0, \bar{L}_0 but different central charges. Nonetheless, one can define an effective central charge for the system by summing over the “strip-wise” $2d$ CFT descriptions.

For the far from BPS case, however, we have a different situation; the conjectured $2d$ CFT description corresponds to a set of D3 giants which have a deformed shape and as a result only certain degrees of freedom on the giant theory survive our (“ $\alpha' \rightarrow 0$ ”) decoupling limit. In a sense, instead of intersecting giants of the near-BPS case, at the extremal point ($\mu = \mu_c$) we are dealing with a (non-marginal) bound state of giants. This could also be traced in the corresponding $6d$ gravity theory obtained from reduction of $10d$ IIB theory (see appendix A). As discussed, the two species of the intersecting giants in the $6d$ language appear as strings which are either electrically and/or magnetically charged under the three-form F_3 . The bound state of giants in the $6d$ theory is then expected to appear as a usual “ (Q_e, Q_m) -string”. The mass of this dyonic (Q_e, Q_m) -string state can be computed working out the time-time component of the energy momentum tensor of the system T_0^0 for the $AdS_3 \times S^3$ configuration. This has two parts, a cosmological constant piece and the part which involves the three-form charges. The latter can be used to identify the mass squared of the (Q_e, Q_m) -string, which is

$$M_{(Q_e, Q_m)}^2 = T_s^{(6)} (N_e^2 \mathbf{g}_s + N_m^2 \mathbf{g}_s^{-1}) \tag{7.1}$$

where $\mathbf{g}_s = \langle X^{-2} \rangle$ is the “effective” $6d$ string coupling and N_e, N_m are the number of electric and magnetic strings and are related to Q_e, Q_m as (4.13).²¹

To complete this picture one should in fact show that the $6d$ (Q_e, Q_m) -string discussed in section 4 is indeed a BPS, stable configuration in the corresponding gravity theory. Moreover, it is plausible to expect that our $6d$ gravity description is a part of a new type of $6d$ gauged supergravity. This $6d$ theory is expected to be a $U(1)^2$ $\mathcal{N} = (1, 1)$ gauged SUGRA with the matter content (in the language of $6d$ $\mathcal{N} = 1$): one gravity multiplet, one tensor multiplet and two $U(1)$ vector multiplets. This theory is nothing but a $6d$ version of the $d = 4, d = 5$ “gauged STU” models (e.g. see [4, 5]) and may be obtained from a suitable generalization of the reduction we already discussed in appendix A. The two $U(1)$ gauge fields A_i are coming from replacing $d\chi_i$ in reduction ansatz (A.1) with $d\chi_i + LA_i$. The details of this reduction and construction and analysis of this “ $6d$ gauged STU” supergravity will be discussed in an upcoming publication [27].

²¹Note that with the action (4.1) we are working in “Einstein frame” and the mass of fundamental string mass squared is $T_s^{(6)} \mathbf{g}_s$.

In section 6 we gave a description of both the near-BPS and far from BPS cases in terms of specific sectors of large R -charge, large engineering dimension operators. We expect these sectors to be decoupled from the rest of the theory since they also have a description in terms of a unitary $2d$ CFT. The near-BPS case has features similar to the BMN sector. In this case, however, the sector is identified with operators of $J_i \sim N^{3/2}$, as opposed to $J \sim N^{1/2}$ of BMN case. In the far from BPS case the operators we are dealing with are far from being BPS and their R -charge J_i ($i = 2, 3$) scale as N^2 . Understanding these sectors in the $4d$ gauge theory and computing their effective 't Hooft expansion parameters, namely effective 't Hooft coupling and the planar-nonplanar expansion ratio, is an interesting open question. From our analysis, however, we expect there should be new “double scaling limits” similarly to the BMN case. It is also desirable to give another supportive evidence for the decoupling of these sectors by counting degeneracy of the states in both of these sectors in $\mathcal{N} = 4$ SYM and matching it with the Bekenstein-Hawking entropies computed here.

Here we focused on the two-charge $5d$ extremal black hole solutions of $U(1)^3$ $5d$ gauged supergravity. The $U(1)^4$ $d = 4$ gauged supergravity has a similar set of black hole solutions [4, 5, 38]. Among them there are three-charge extremal black holes. One can take the near-horizon decoupling limits over these black holes to obtain $AdS_3 \times S^2$ geometries. Again there are two possibilities, the near-BPS and far from BPS cases, very much the same as what we found here in the $5d$ case. Detailed analysis of these decoupling limits is what we present in an upcoming work [39].

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A. Reduction $10 \rightarrow 6$

Here we present some of the details of the computations for the reduction of $10d$ IIB supergravity to the $6d$ gravity theory discussed in section 4. We start with the following reduction ansatz for the $10d$ metric:

$$ds_{(10)}^2 = \mu_1 ds_{(6)}^2 + \frac{L^2}{\mu_1} \sum_{i=2,3} X_i^{-1} (d\mu_i^2 + \mu_i^2 d\chi_i^2) \tag{A.1}$$

where χ_i range over $[0, 2\pi]$,

$$\mu_1^2 = 1 - \mu_2^2 - \mu_3^2, \tag{A.2}$$

and $ds_{(6)}^2$ is the $6d$ metric, x denotes the $6d$ coordinates;

$$X_2 X_3 = 1, \quad X_2(x) \equiv X(x) \tag{A.3}$$

and $X(x)$ is the $6d$ scalar coming from the reduction. As it is seen from (A.2), $0 \leq \mu_i \leq 1$, $i = 2, 3$. In what follows we will use Capital Latin indices M, N, P, \dots for $10d$

coordinates, little Latin indices i, j, k, \dots for the four-dimensional reduction manifold and Greek indices $\mu, \nu, \dots = 0, 1, \dots, 5$ for the $6d$ spacetime.

In the absence of the scalar field, i.e. when $X(x) = 1$, the reduction manifold \mathcal{M}_4 is simply a four-dimensional ball of radius L and hence is not “compact” in the topological sense. Moreover, the $10d$ metric for this case has a curvature singularity at $\mu_1 = 0$. We expect this singularity to be removed once the stringy corrections are considered. The volume of the \mathcal{M}_4 is

$$V_{\mathcal{M}_4} = (2\pi)^2 L^4 \int \mu_2 \mu_3 d\mu_2 d\mu_3 = \frac{\pi^2}{2} L^4. \quad (\text{A.4})$$

Therefore the Newton constant of the $6d$ theory we are going to derive is

$$G_N^{(6)} = \frac{G_N^{(10)}}{V_{\mathcal{M}_4}} = \frac{G_N^{(10)}}{\frac{\pi^2}{2} L^4}. \quad (\text{A.5})$$

Besides the reduction ansatz for the metric we also need to give the reduction ansatz for the other fields of the $10d$ IIB theory. In our case we choose to turn off all the form fields and the dilaton, except for the (self-dual) five-form of the IIB theory which is reduced such that leads to a three-form F_3 in $6d$. Explicitly,

$$F_5 = X(\mathcal{J}_+ \wedge F_{3+} + \mathcal{J}_- \wedge F_{3-}), \quad (\text{A.6})$$

$$\mathcal{J}_\pm = \frac{1}{X} d\mu_2^2 \wedge d\chi_2 \pm X d\mu_3^2 \wedge d\chi_3, \quad (\text{A.7})$$

$$F_{3\pm} = \frac{1 \pm *_6}{2} F_3. \quad (\text{A.8})$$

where \mathcal{J}_\pm (and $F_{3\pm}$) are the self-dual and anti-self-dual two-form (three-form) fields on the \mathcal{M}_4 (in $6d$) and $\mathcal{J}_+ \wedge \mathcal{J}_-$ is its volume form. From the above reduction ansatz for five-form it is evidently seen that F_5 is self-dual.

To show that the above reduction ansatz for metric and the five-form really leads to a consistent reduction of IIB theory to a six-dimensional theory we need to work at the level of equations of motion and show that set of $10d$ IIB equations of motion lead to a consistent system of equations for a $6d$ gravity theory coupled to a scalar X and a three-form F_3 . The IIB equations of motion relevant to our case are (e.g. see [5])

$$\text{e.o.m for metric:} \quad R_{MN} = \frac{1}{96} (F_5^2)_{MN}, \quad (F_5^2)_{MN} \equiv F_{M P_1 P_2 P_3 P_4} F_N^{P_1 P_2 P_3 P_4}, \quad (\text{A.9})$$

$$\text{e.o.m for five-form:} \quad F_5 = *F_5, \quad dF_5 = 0. \quad (\text{A.10})$$

It is notable that, as a result of self-duality of the five-form

$$F_5^2 = F_{P_1 P_2 P_3 P_4 P_5} F^{P_1 P_2 P_3 P_4 P_5} = 0,$$

and hence the equation of motion for metric also implies $R = R_{MN} g^{MN} = 0$, which in turn is the equation of motion for constant dilaton for our ansatz.

Writing (A.6) in components,

$$F_5 = \frac{1}{3!} F_{3\mu\nu\rho} d\mu_2^2 \wedge d\chi_2 \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho + \frac{1}{3!} X^2 (*F_3)_{\mu\nu\rho} d\mu_3^2 \wedge d\chi_3 \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho, \quad (\text{A.11})$$

it is evidently seen that the five-form equation of motion, $dF_5 = 0$ implies the equations of motion for the three-form:

$$dF_3 = 0 \quad (\text{A.12})$$

$$d(X^2 * F_3) = 0. \quad (\text{A.13})$$

The metric equations of motion, decomposes into three independent set of equations; the $R_{\mu_2\mu_2}$, $R_{\mu_3\mu_3}$ and the $R_{\mu\nu}$ components. Computing the $10d$ Ricci tensor with the ansatz (A.1) we obtain

$$R_{\mu_2\mu_2} = g_{\mu_2\mu_2}^{(10)} \frac{1}{\mu_1} \left(\frac{\Delta X}{2X} - \frac{\nabla X^2}{2X^2} + \frac{1}{L^2} (X - X^{-1}) \right) \quad (\text{A.14a})$$

$$\frac{R_{\mu_2\mu_2}}{g_{\mu_2\mu_2}^{(10)}} = -\frac{R_{\mu_3\mu_3}}{g_{\mu_3\mu_3}^{(10)}} \quad (\text{A.14b})$$

$$R_{\mu\nu}^{(10)} = R_{\mu\nu}^{(6)} + \frac{1}{L^2} (X + X^{-1}) g_{\mu\nu}^{(6)} - \frac{1}{X^2} \nabla_\mu X \nabla_\nu X. \quad (\text{A.14c})$$

The right-hand-side of the metric equations of motion is also computed as

$$(F_5^2)_{\mu_2\mu_2} = g_{\mu_2\mu_2}^{(10)} \frac{16X^2}{\mu_1} F_{3\mu\nu\rho} F_3^{\mu\nu\rho} \quad (\text{A.15a})$$

$$(F_5^2)_{\mu_3\mu_3} = g_{\mu_3\mu_3}^{(10)} \frac{16X^2}{\mu_1} (*F_3)_{\mu\nu\rho} (*F_3)^{\mu\nu\rho} \quad (\text{A.15b})$$

$$(F_5^2)_{\mu\nu} = 48X^2 \left(F_{3\mu\rho\lambda} F_{3\nu}^{\rho\lambda} + (*F_3)_{\mu\rho\lambda} (*F_3)_{\nu}^{\rho\lambda} \right). \quad (\text{A.15c})$$

Recalling that

$$\begin{aligned} F_{3\mu\rho\lambda} F_{3\nu}^{\rho\lambda} &= (*F_3)_{\mu\rho\lambda} (*F_3)_{\nu}^{\rho\lambda} + \frac{1}{3} g_{\mu\nu}^{(6)} F_{3\alpha\rho\lambda} F_3^{\alpha\rho\lambda}, \\ F_{3\mu\rho\lambda} F_3^{\mu\rho\lambda} &= -(*F_3)_{\mu\rho\lambda} (*F_3)^{\mu\rho\lambda}, \end{aligned} \quad (\text{A.16})$$

We see that the $\mu_2\mu_2$ and $\mu_3\mu_3$ are consistent and become identical. (The $\chi_i\chi_i$ components are hence identical too.) This proves the consistency of our reduction ansatz.

In sum, the $10d$ equations of motion are all satisfied if $6d$ metric $g_{\mu\nu}^{(6)}$, X and the three-form satisfy

$$\begin{aligned} R_{\mu\nu}^{(6)} &= \frac{1}{X^2} \nabla_\mu X \nabla_\nu X - \frac{1}{L^2} (X + X^{-1}) g_{\mu\nu}^{(6)} \\ &\quad + X^2 \left(F_{3\mu\rho\lambda} F_{3\nu}^{\rho\lambda} - \frac{1}{6} g_{\mu\nu}^{(6)} F_{3\alpha\rho\lambda} F_3^{\alpha\rho\lambda} \right) \end{aligned} \quad (\text{A.17a})$$

$$\frac{\Delta X}{2X} - \frac{\nabla X^2}{2X^2} + \frac{1}{L^2} (X - X^{-1}) = \frac{X^2}{6} F_{3\mu\rho\lambda} F_3^{\mu\rho\lambda} \quad (\text{A.17b})$$

$$d(X^2 * F_3) = 0, \quad dF_3 = 0 \quad (\text{A.17c})$$

The above equations of motion can be obtained from the $6d$ gravity action

$$\mathcal{S}_6 = \frac{1}{16\pi G_N^{(6)}} \int d^6x \sqrt{-g^{(6)}} \left[R^{(6)} - g^{\mu\nu} \frac{\nabla_\mu X \nabla_\nu X}{X^2} + \frac{4}{L^2} (X + X^{-1}) - \frac{X^2}{3} F_{3\mu\nu\rho} F_3^{\mu\nu\rho} \right]. \quad (\text{A.18})$$

To bring the kinetic term into the canonical form one may define the scalar field ϕ as

$$X = e^\phi$$

in which case the potential becomes $8 \cosh \phi/L^2$.

B. Conventions for rotating BTZ and conical spaces

In this appendix we give a brief review of the definitions of all possible stationary $3d$ locally AdS_3 spacetimes, obeying $R_{\mu\nu} = -\frac{2}{R^2}g_{\mu\nu}$. The most generic solution is of course the BTZ-type black hole

$$ds^2 = R^2 \left\{ -\frac{r^4 + 2(a_+^2 + a_-^2)r^2 + (a_+^2 - a_-^2)^2}{r^2} dt^2 + \frac{r^2}{r^4 + 2(a_+^2 + a_-^2)r^2 + (a_+^2 - a_-^2)^2} dr^2 + r^2 \left(d\phi + \frac{a_+^2 - a_-^2}{r^2} dt \right)^2 \right\}, \quad (\text{B.1})$$

where $\phi \in [0, 2\pi]$. Without loss of generality we can always assume $a_+^2 \leq a_-^2$. It is useful to introduce to other parameters

$$a_+^2 = -\frac{M+J}{4}, \quad a_-^2 = -\frac{M-J}{4}, \quad J \geq 0 \quad (\text{B.2})$$

We are then left with the following three possibilities (e.g. see [32])

- $a_+^2, a_-^2 > 0$, corresponding to $M < -J$. In this case we are generically dealing with a space with conic singularity. The special case of $a_+ = a_- = 1/2$ corresponds to a *global* AdS_3 . For the generic case $a_+ = a_- = \gamma/2$, where $J = 0$, the conic space has the same line element as a global AdS_3 but its ϕ coordinate is now ranging over $[0, 2\pi\gamma]$. In string theory for *rational* values of γ and only when $\gamma < 1$ the conical singularity could be resolved [23]. For the general case when $a_+ \neq a_-$, the conical space can be resolved in string theory only when a_-^2 is a rational number and $0 \leq a_-^2 \leq 1/4$ (we are assuming that $a_-^2 \geq a_+^2$ and that $J \in \mathbb{Z}$). In terms of M, J that is

$$-1 \leq M - J \equiv -\gamma^2 < -2J, \quad \gamma \in \mathbb{Q}, \quad J \in \mathbb{Z}. \quad (\text{B.3})$$

- $a_+^2 < 0, a_-^2 > 0$, corresponding to $-J < M < J$. The geometry is ill-defined and cannot be made sense of in string theory.
- $a_+^2, a_-^2 \leq 0$, corresponding to $M \geq J$, defines a rotating BTZ black hole of mass M and angular momentum J [29]. For this case the black hole temperature (measured in units of R) is

$$T_{\text{BTZ}} = \frac{\sqrt{M^2 - J^2}}{2\pi\rho_h}, \quad \rho_h = \frac{1}{2} \left(\sqrt{M+J} + \sqrt{M-J} \right). \quad (\text{B.4})$$

The special case of $a_- = a_+$ (that is the $J = 0$ case) corresponds to static BTZ black hole. The $a_- = 0$ (or equivalently $M = J$) case corresponds to extremal rotating BTZ which has zero temperature and finally the very special case of $a_- = a_+ = 0$ corresponds to massless BTZ black hole.

To summarize the above, the cases with integer-valued J and when $M - J \geq -1$ are those which are sensible geometries in string theory. For the $-1 < M - J < 0$ resolution of conical singularity in string theory also demands $\sqrt{J - M}$ to be a rational number.

As has been discussed in [23, 36, 40, 41] among the above cases $M \leq -J$ for any M, J and $M = J, M \geq 0$ can be supersymmetrized. For the $M \leq -J$ case the solution becomes supersymmetric in a $3d$ gauged supergravity which has at least two $U(1)$ gauge fields. In our case to maintain supersymmetry one then needs to turn on the Wilson lines of both of the $U(1)$ (flat-connection) gauge fields. The two gauge fields which make the above metric supersymmetric are then [23, 36, 40]

$$A^{(1)} = a_+(dt + d\phi), \quad A^{(2)} = a_-(dt - d\phi), \quad (\text{B.5})$$

where $A^{(1)}, A^{(2)}$ are the flat connections of the two $U(1)$'s. For $M = J, M \geq 0$ no gauge fields are needed to keep supersymmetry. Among the supersymmetric configurations the *global* AdS_3 , that is when $a_+ = a_- = 1/2$ keeps the maximum supersymmetry the $3d$ theory has, with *anti-periodic* boundary conditions on fermions on the ϕ direction. The massless BTZ case, that is when $a_+ = a_- = 0$, as well as the extremal BTZ (corresponding to $a_+^2 = a_-^2 > 0$) keep half of the maximal supersymmetry but with *periodic* boundary conditions on the fermions on the ϕ direction [32]. The conical spaces also keep half of maximal supersymmetry.

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